

Numerical design method for conic curved Fresnel lens

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Abstract: Fresnel lens is one of the most common solar concentrators. The performance of the curved Fresnel lens is usually better than flat Fresnel lens. Based on the principle of non-imaging optics, a new curved Fresnel lens design method was proposed in this study. The curved Fresnel lens was on conical surface. In the premise of meeting the mechanical requirements of the Fresnel lens, the slope of the second surface of the Fresnel lens was solved. Based on the objective of manufacturability, different shapes of Fresnel lenses were obtained by this method to analyze the effect of structural parameters of curved Fresnel lens on concentration ratio, acceptance angle and the illumination uniformity by optical simulation. The effect of machining error on optical efficiency in ultra-precision machining was also analyzed. The design method provided a new way for the parametric analysis of curved Fresnel lens. The simulation results show that the small aspect ratio of curved Fresnel lens will get good uniformity and high energy efficiency.

Key words: non-imaging optics; solar concentrator; curved Fresnel lens; ultra-precision turning

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圆锥曲面 Fresnel 透镜数值设计方法

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摘要: Fresnel 透镜是最常见的太阳能聚光镜之一, 曲面 Fresnel 透镜通常比平面 Fresnel 透镜具有更加优越的性能。基于非成像光学理论, 提出了一种新型曲面 Fresnel 透镜的设计方法。曲面 Fresnel 透镜采用圆锥曲面基底, 在满足 Fresnel 透镜机械参数要求的前提下, 实现 Fresnel 透镜第二工作面面型的求解。基于可加工性的要求, 利用该方法设计了不同形状的曲面 Fresnel 透镜, 通过光学仿真分析了曲面 Fresnel 透镜的结构参数对聚光镜会聚比、容忍角、光照均匀性的影响, 并分析了超精密加工条件下, 由于加工误差对光学效率的影响。该设计方法为参数化曲面 Fresnel 透镜的分析提供了一种新的途径。仿真结果表明, 具有小深宽比的曲面 Fresnel 透镜具有更好的均匀性和更高的能量利用率。

关键词: 非成像光学; 太阳能聚光镜; 曲面 Fresnel 透镜; 超精密车削

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0 Introduction

Concentrated photovoltaic(CPV) systems use large area of concentrators to converge sunlight on small solar cells, which can reduce the area of solar cells and consequently reduce environmental pollution caused by solar cells production^[1]. The solar concentrator is the primary part of a CPV system. The concentrator can be employed to realize the collection of light and the uniformity of energy. Fresnel lens is one of the most common solar concentrators^[2-3], which is divided into two types: flat lens and curved lens.

Many researchers have studied the curved Fresnel lens. The most common substrate of curved Fresnel lens is spherical substrate^[4]. O'Neil et al. developed a type of Fresnel lens for space power system^[5]. The purpose of the design was to achieve ultra-thin requirement. The Fresnel lens also was on a spherical surface, and the incident plane of each prism unit was approximated as an oblique plane. Yeh et al. designed an elliptical-based Fresnel lens concentrator^[6]. The influence of elliptical parameters on concentrator performance was not analyzed. Leutz et al. proposed a design and optimization of a convex shaped nonimaging Fresnel lens^[7], which is a three-dimensional design method within two design angles to the absorber. But the Fresnel lens mainly used for the solar thermal collector, and had low concentration ratio. With the development of freeform design method, the Fresnel lens with freeform surface base shape began to appear^[8]. Languy et al. developed the shaped achromatic Fresnel lenses^[9], which can be used to enhance the concentration ratio and to achieve uniform flux on the solar cell.

In this paper, a new method based on discretization of curved Fresnel lens design is proposed. The method is flexible and accurate, which can realize any conical surface shaped Fresnel lens design for the special mechanical requirements on the Fresnel lens. In addition, the Fresnel lens models can be built by this

method to analyze the parameters of the Fresnel lens on the optical performance, which provides a modeling method for the parametric analysis of curved Fresnel concentrator.

1 Design method

The Fresnel lens surface composed of discrete points which are obtained by using geometric construction method (GCM)^[10]. There are two working surfaces of the Fresnel lens. The first working surface is known as a conical surface, and the second working surface is unknown and needs to be solved.

The equation of conic section is:

$$y=f+d-\frac{cx^2}{1+\sqrt{1-(1+k)c^2x^2}} \quad (1)$$

Where, f is focal length, d is thickness of lens, k is the conic constant, c is the curvature, x is distance (radius) from center of lens, y is depth.

The incident ray is traveling in a medium of refractive index n_1 in a direction defined by the unit vector u , the normal vector of the surface is n . The refracted ray travels in a medium of refractive index n_2 in a direction defined by the unit vector r .

$$p_1=n_1u \quad (2)$$

$$p_2=n_2r \quad (3)$$

Where, p_1 and p_2 are the optical momenta of the ray before and after refraction, which satisfied Eq.(4), and where $\|u\|=\|r\|=1$. The unit vector n can be solved by Eq.(5).

$$p_2=p_1-(p_1 \cdot n)n+\sqrt{n_2^2-n_1^2+(p_1n)^2}n \quad (4)$$

$$n=\frac{p_1-p_2}{\|p_1-p_2\|} \quad (5)$$

Eq.(4) and Eq.(5) are the vector form of Snell's Law^[11].

The curved Fresnel lens is composed of a series of lens units, and the angle between the two edges of each lens unit to the center of the cell is equal. A cartesian coordinate system is established with the origin at the center of the cell and with the symmetry axis of the conic section as coordinate axis Y , as shown in Fig.1. The calculation flow chart of curved

Fresnel lens is shown in Fig.2.

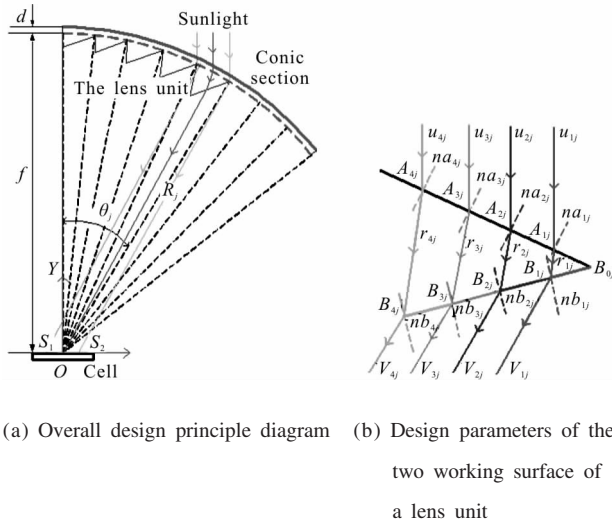


Fig.1 Design principle of the curved Fresnel lens

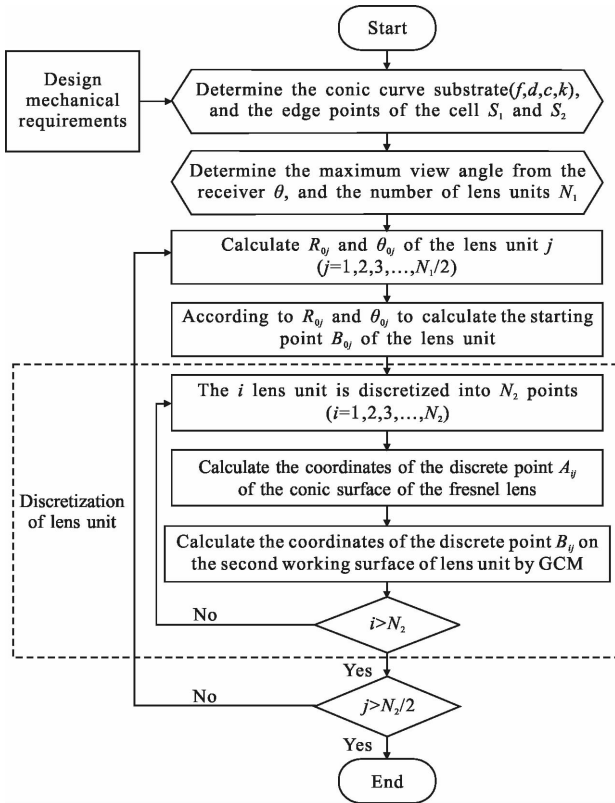


Fig.2 Calculation flow chart of curved Fresnel lens

The maximum view angle from the cell is θ , the number of lens units is N_1 , so the angle between the edge of each lens unit is θ/N_1 .

Before calculating, the known parameters need to be clarified. The substrate equation (f, d, c, k), the

maximum view angle from the receiver θ , the incident ray u_{ij} , the position and size of receiver, the number of units of Fresnel lens N_1 are known. The points on the two valid working surface are represented by A_{ij} and B_{ij} respectively, A_{ij} represents the coordinates of the discrete points i of the lens unit j .

Firstly, calculate the initial point coordinates of the second surface of each unit B_{0j} . The conic section is discretized by Eq.(6), the edge point of lens unit $A_{0j}(x_{0j}, y_{0j})$ can be obtained.

$$\begin{cases} x_{0j}=R_{0j}\sin\theta_{0j} \\ y_{0j}=R_{0j}\cos\theta_{0j} \end{cases} \quad (6)$$

Where, θ_{0j} is the angle between the edge of lens unit j and axis Y ($j=1, 2, \dots, N_1/2$). R_{0j} is the distance from O to edge points of the lens unit j of conical surface.

Then, each lens unit is dispersed. The point $A_{ij}(x_{aj}, y_{aj})$ on the conical surface satisfies Eq.(7), R_{ij} is the distance between $A_{ij}O$ and axis Y ($i=1, 2, \dots, N_2$).

$$\begin{cases} x_{aj}=R_{ij}\sin\theta_{ij} \\ y_{aj}=R_{ij}\cos\theta_{ij} \end{cases} \quad (7)$$

The normal vector na_{ij} of A_{ij} satisfies Eq.(8), and u_{ij} is the unit vector of incident ray.

$$na_{ij}=\begin{bmatrix} cx_{aj}[2\sqrt{1-c^2x_{aj}^2-c^2x_{aj}^2k}+2-c^2x_{aj}^2(1+k)] \\ (1+\sqrt{1-c^2x_{aj}^2-c^2x_{aj}^2k})^2\sqrt{1-c^2x_{aj}^2-c^2x_{aj}^2k} \end{bmatrix}^T \quad (8)$$

The ray r_{ij} is the incident ray of the second surface of Fresnel lens, which can be obtained by Eq.(9) and Eq.(10).

$$r_{ij}=u_{ij}+(\sqrt{n^2-1+\cos I}-\cos I)na_{ij} \quad (9)$$

$$\cos I=\frac{u_{ij} \cdot na_{ij}}{\|u_{ij}\| \|na_{ij}\|} \quad (10)$$

Last, B_{ij} and the normal vector nb_{ij} of the second surface are calculated on the basis of the GCM. The Snell's Law can be established on the two surfaces of Fresnel lens according to Eq.(2)–(5). The straight line element $\overline{B_{i-1}B_{ij}}$ is perpendicular to the normal vector nb_{ij} , which satisfies Eq.(11).

$$E_{ij}=\overline{B_{i-1}B_{ij}} \quad (11)$$

$$E_{ij} \cdot nb_{ij}=0 \quad (12)$$

In addition, the design of Fresnel lens with

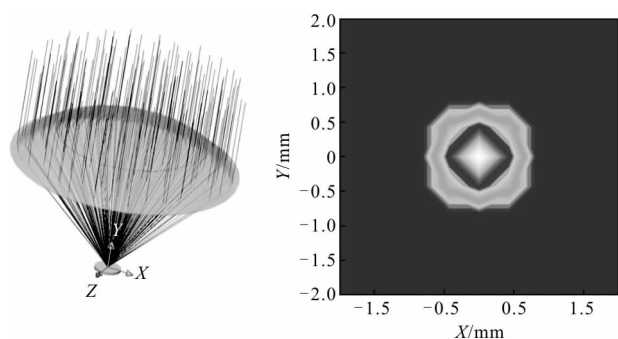
arbitrary spot size can be accomplished by GCM, and design principle is shown in Fig.1. S_1 and S_2 are the edge points of the cell. The calculations are the same as above except that the exit ray is

$$v_{ij}=S_i-B_{ij} \quad (13)$$

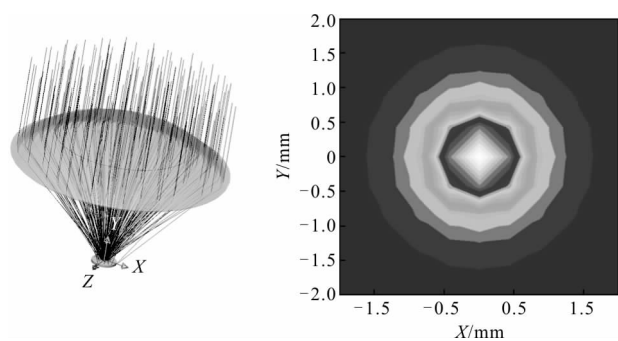
Where, the refracted ray is v_{ij} .

2 Modeling and ray tracing results

The method can be used to design arbitrary conic Fresnel lenses programmatically. Different models can be built to analyze the effect of the parameters on concentrator performance in order to realize the optimization of Fresnel lens design. The material of Fresnel lens is PMMA. According to the relationship between the refractive index and wavelength^[12], the reference wavelength (537 nm, the refractive index is 1.493 5) is selected by the optical simulation under AM1.5. The model is established and the optical simulation is carried out by one wavelength (537 nm) and solar spectrum respectively is shown in Fig.3. The parameters of the model: $f=60$ mm, $d=0.5$ mm, $\theta=90^\circ$, $c=1/30$ mm⁻¹, $k=-0.5$, $N_1=32$.



(a) Only one wavelength (537 nm)



(b) Solar spectrum

Fig.3 Ray tracing simulation by two different sources

3 Parameters analysis

The important parameters for evaluating the performance of concentrators involve concentration ratio, optical efficiency^[13], acceptance angle^[14], the illumination uniformity on cells. Because the method can be employed to achieve the design of Fresnel lenses with different parameters, it provides a way of parameters analysis of curved Fresnel lens.

3.1 Concentration ratio

The concentration ratio affects the conversion efficiency and life time span of photovoltaic cell directly. The concentration ratio means the geometric concentration ratio in this paper^[11]. The effect of the conic section parameters c and k on concentration ratio is analyzed in Fig.4, the radius of the cell is 2 mm. As can be seen from Fig.4, curved Fresnel lens with a small conic constant and negative curvature would have a higher concentration ratio.

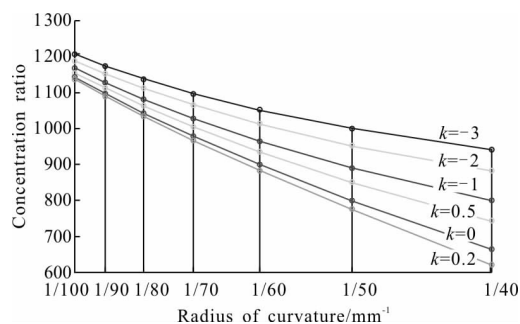


Fig.4 Relation between conic parameters and concentration ratio, $f=60$ mm, $d=2$ mm, $\theta=90^\circ$

3.2 Optical efficiency

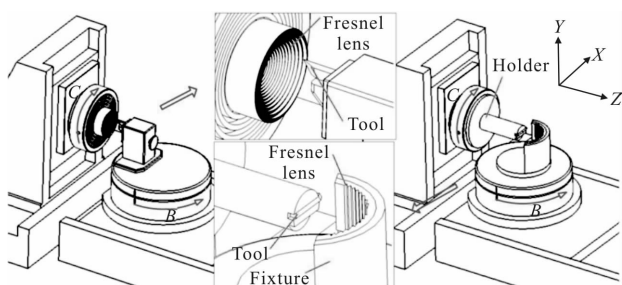
The optical efficiency of the lens is defined^[15]:

$$\eta = \frac{\sum_{i=1}^k \tau_i L_i (1 - L_{tipk} - L_{blockk} - L_{missk})}{\sum_{i=1}^k L_i} \quad (14)$$

Where, k is the number of lens units, τ_i is the transmittance including reflectance losses, L_i is the effective width of the unit lens, L_{tipk} is tip losses, L_{blockk} is blocking losses, L_{missk} is absorber misses.

The ultra-precision turning is employed to get the

curved Fresnel lens. The rotational symmetry curved Fresnel lens is using conventional ultra-precision turning, and the cylindrical Fresnel lens is using the flying cutting, the schematic diagram is shown in Fig.5. The flying cutting uses special Fresnel tool with radius of 0.02 mm. There is a burr area of 3 μm to 5 μm at the top of the lens unit. This processing method can achieve 0.005° angle error. These errors will reduce the optical efficiency. Taking the influence brought by manufacturing in consider, set the width of the lost light of each lens unit is 25 μm.



(a) Rotational symmetry Fresnel lens (b) Cylindrical Fresnel lens

Fig.5 Schematic diagram of the ultra-precision machining of curved Fresnel lens

The transmittance of the two surface of the lens is shown in Eq.(16), the angles θ_1 and θ_2 refer to the angle of incidence and the angle of refraction. The relationship between conic parameters and optical efficiency is shown in Fig.6, the greater k , the greater c , the higher the optical efficiency.

$$\tau_k = \tau_{\rho 1} \tau_{\rho 2} \quad (15)$$

$$\tau_{\rho} = 1 - \frac{1}{2} \left[\frac{\tan^2(\theta_1 - \theta_2)}{\tan^2(\theta_1 + \theta_2)} + \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} \right] \quad (16)$$

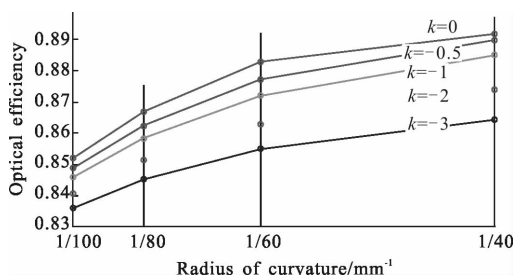


Fig.6 Relation between conic parameters and optical efficiency,

$f=60$ mm, $d=2$ mm, $\theta=90^\circ$, $N_1=40$

The effect of the number of lens units on the optical efficiency is shown in Fig.7, $f=60$ mm, $d=2$ mm, $c=1/60$ mm⁻¹, $k=-1$, $\theta=90^\circ$. It is seen that the more the number of lens units, the smaller the optical efficiency. This is because the greater the number of units, the greater the error caused by manufacturing.

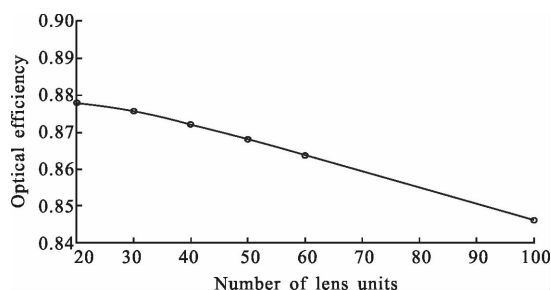


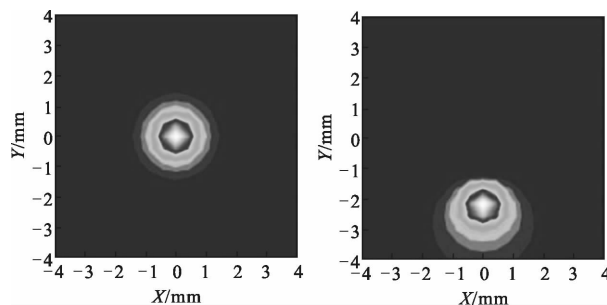
Fig.7 Relation between the number of lens units and optical efficiency

3.3 Acceptance angle

The acceptance angle is the angle between the concentrator axis and the sunlight, and acceptance angle is inversely proportional to concentration ratio. Large acceptance angle will relax the optical requirement and the precision of the tracker of CPV system, and reduce the cost of power generation.

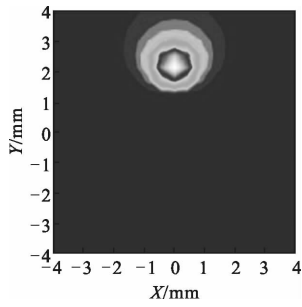
In the case of spot focusing curved Fresnel lens, when the incident angle is 0°, 2°, -2° respectively, the spot position is shifted. The practical limit of the spot as shown in Fig.8, $f=60$ mm, $d=0.5$ mm, $c=1/30$ mm⁻¹, $k=-0.5$, $\theta=90^\circ$.

The increased acceptance angle comes at the expense of decreased concentration ratio. The relationship between acceptance angle and concentration ratio is shown in Fig.9. Low concentration solar concentrator has greater acceptance angle.



(a) Incident angle is 0°

(b) Incident angle is 2°



(c) Incident angle is -2°

Fig.8 Simulation irradiance on cell of different acceptance angles

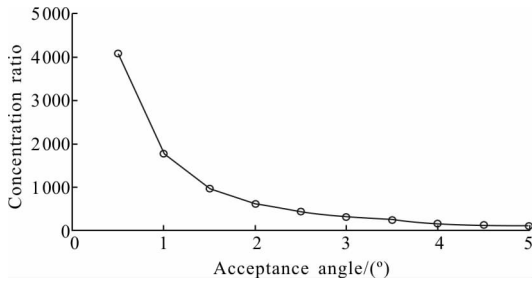


Fig.9 Relation between acceptance angle and concentration ratio

3.4 Illumination uniformity on solar cell

This paper analyzes the illumination uniformity on the cylindrical Fresnel lens. Six models are established, and the parameters are shown in Tab.1. The width of the receiver is 2 mm. The shape of the models is shown in Fig.10. The optical simulation of the six models is carried out by using solar spectrum, and the optical simulation results are given in Fig.11.

Tab.1 Parameters of the six Fresnel lenses

Number	f/mm	$\theta/(\text{^\circ})$	c/mm^{-1}	k	ar
①	60	100	1/80	-0.5	2.220 0
②	60	120	1/60	-0.5	2.108 8
③	60	90	1/60	-0.5	2.038 6
④	80	90	1/60	-0.5	1.885 6
⑤	60	90	1/60	1	1.885 6
⑥	80	90	1/30	-0.5	1.414 2

The aspect ratio ar is shown in Eq.(17), D is aperture of the lens.

$$ar = \frac{D}{f+d} \quad (17)$$

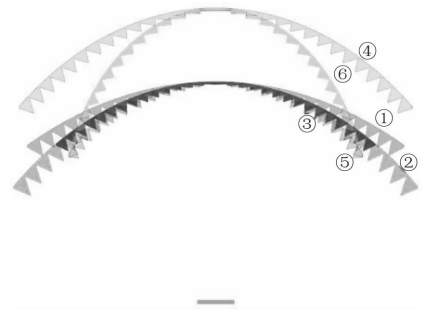


Fig.10 Shape of the six Fresnel lenses

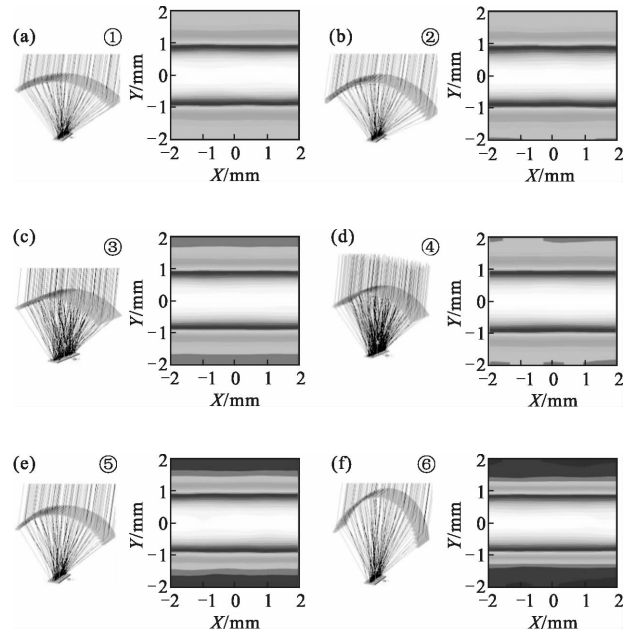


Fig.11 Illumination distribution of the six curved Fresnel lenses

From the simulation results, the uniformity is better in the range of -1 mm to 1 mm . But due to the use of the solar spectrum, the material has dispersion which will affect the actual width of the spot. These will cause energy loss and uneven distribution of energy on receiver. Aspect ratio and the curved surface shape are the important factors that affect the illumination uniformity. Aspect ratio is determined by focal length of lens and conical surface parameters. When aspect ratio is too large and the curved surface is too steep, some light is lost due to total reflection. Therefore, in precondition of curved shape satisfying design requirement, the small aspect ratio of lens is used which will get a good uniformity and high energy efficiency.

4 Conclusions

In this paper, a freeform surface design method is proposed and applied to design the curved Fresnel lens, which can realize the design of arbitrary conic shape curved Fresnel lens programmatically. The effects of the design conic parameter c and k on concentration ratio, optical efficiency, acceptance angle, and the illumination uniformity on the cell are analyzed by means of numerical calculation and optimal curved Fresnel lens parameters. The simulation results show that curved Fresnel lens with a small conic constant and negative curvature would have a higher concentration ratio, and the small aspect ratio of lens will get a good uniformity and high energy efficiency.

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