Analysis on tracking convergent property of reticle seeker

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Abstract: The tracking convergent property is an important guideline for a reticle seeker. The tracking of the reticle seeker is driven by the period mean torque which is generated by the tracking signal. In this paper, the convergent property of the reticle seeker was analyzed and some significant conclusions were attained using series theory. The convergence of the tracking procession was affected by both the tracking coefficient and the phase bias $\Delta\theta$ between the target location and the tracking signal. The convergent conditions and the convergent step function were also obtained. These conclusions are important for the design of the convergent tracking algorithm of the reticle seeker.

Key words: tracking convergence; period mean torque; reticle seeker

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回元导引头跟踪收敛性研究

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摘 要:位标器的跟踪性能由跟踪信号驱动形成的周期平均力决定,跟踪收敛性是位标器跟踪性能的重要体现。应用级数理论分析了位标器的跟踪收敛特性,得出了位标器的跟踪收敛性受到位标器跟踪系数 k 和跟踪相位误差 $\Delta\theta$ 影响的结论。并通过计算给出了位标器跟踪的收敛条件和收敛步长,即随着跟踪系数 k 与 1 的偏差 Δk 及跟踪相位误差 $\Delta\theta$ 增大,跟踪收敛步长也会相应增大。这些结论会给位标器跟踪算法的设计提供重要的理论支撑。

关键词:跟踪收敛性; 周期平均力; 位标器

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0 Introduction

The infrared detecting and tracking system which includes the reticle seeker and the electrical control unit is widely used in surface to air and air to air missiles. The reticle seeker modulates the incoming light signals of the target radiation with the rotating of the gyro and then changes the modulated light signal into the electrical signal by the optical subsystem. The electrical control unit samples and processes the electrical signal in order to calculate the location of the target.

Based on the target location, the sinusoidal tracking signal will be generated to drive the origin of the coordinates fixed on the gyro to the target location, which means the new target location will be zero in the new coordinates. Usually, the frequency of the tracking signal is the same as the modulation frequency, the amplitude of tracking signal is in promotion with the target location and the scale is also tracking coefficient **k**, and the phase of the tracking signal represented the angle of the target location.

In fact, the tracking coefficient will not be always proper and the phase of the tracking signal will not be always accurate and the phase bias $\Delta\theta$ must be taken into consideration, so in this paper we analyze the convergent property of the reticle seeker which can be affected by the two factors **k** and $\Delta\theta$, and obtain some useful conclusions which are significant to design the tracking algorithm[1-3].

1 Tracking principles

The tracking function of the reticle seeker is determined by the tracking coils and the gyro. The tracking signal drives the gyro to track the target by sending sinusoidal current signal to the tracking coils. The tracking coils can be seen as a solenoid and the gyro can be seen as a strip of permanent magnet just as Fig.1 demonstrates. According to the Biot-Savart Law, the current passing through the tracking coils will generate magnetic field through the solenoid, suppose the magnetic flux density is $\vec{\mathbf{B}}$:

$$|\mathbf{B}|=\mu_0 \mathbf{n}\mathbf{I}^* \frac{\mathbf{L}}{\sqrt{\mathbf{L}^2+\mathbf{R}^2}} \tag{1}$$

The direction of $\vec{\mathbf{B}}$ is passing through the solenoid to right, as shown in Fig.1. In equation 1, μ_0 is the magnetic permeability in the vacuum circumstance, **n** is the density of the coils, **I** is the current passing through the solenoid, and **L** is the length of the solenoid and **R** is the radium of the solenoid. The magnetic field $\vec{\mathbf{B}}$ will generate torque on the strip of permanent magnet and if the torque is proper, the gyro will always tracking the target[4-5].

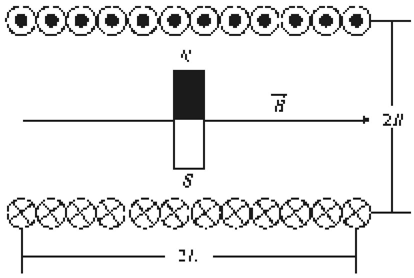

Fig.1 Schemes of tracking unit

For a given seeker, all the parameters mentioned above are explicit except **I**, thus suppose the tracking signal can be descript like this, $\mathbf{I}=\mathbf{I}_0{}^*\cos(\omega t-\theta)$, $\mathbf{I}_0$ is the amplitude of the signal, ω is the frequency of the signal and θ is the initial phase of the signal. Because the gyro is also the modulating unit with the frequency of 100 Hz, it is quite difficult to response the torque produced by $\vec{\mathbf{B}}$ in the instantaneous method because of the fast modulating frequency, but the period mean effects will be obvious. Figure 2 shows the vector map of the torque.

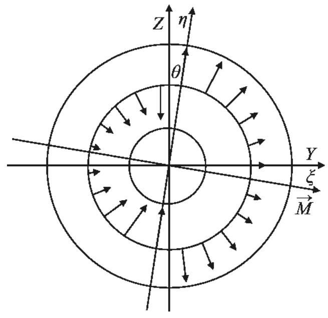

Fig.2 Torque vector map

Supposing $\mathbf{M}_0$ is the rotating torque if only $\mathbf{I}_0$ passing through the tracking coils, then $\mathbf{M}=\mathbf{M}_0*\cos(\omega t-\theta)$ when $\mathbf{I}$ passing throu gh the coils. The absolute value of periods mean torque $\overrightarrow{\mathbf{M}}$ can be obtained like this:

$$|\mathbf{M}|=\frac{1}{\pi}\int_{\theta}^{\pi+\theta}\mathbf{M}_0*\cos(\omega t-\theta)*\cos(\omega t-\theta)d\omega t=\frac{\mathbf{M}_0}{2} \quad (2)$$

The direction of $\overrightarrow{\mathbf{M}}$ is the same as ξ axis in Fig.2.

If $\mathbf{I}_0$ varies in accordance with the target location and θ varies in accordance with the target direction, the proper period mean torque will be used to drive the origin of the coordinate to the target location, which is just the tracking process[6-8].

2 Analysis of the convergent property of reticle seeker

In fact, the tracking coefficient will not be always proper and the phase of the tracking signal must have a bias $\Delta\theta$, thus the tracking signal can be described like this[9-10]:

$$\mathbf{I}_1=\mathbf{k}\mathbf{I}_0*\cos(\omega t-\theta-\Delta\theta) \quad (3)$$

This mean s the period mean torque will have a bias of $\overrightarrow{\Delta\mathbf{M}}$, as demonstrated in Fig.3. The absolute value and the direction are both determined by $\mathbf{k}$ and $\Delta\theta$.

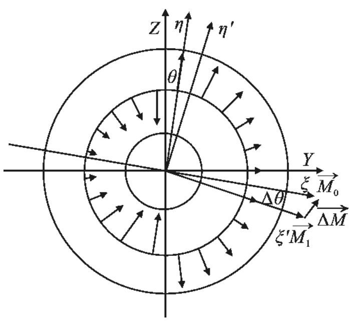

Fig.3 Bias of the period mean force

Suppose the target is a fixed position one and its location is $\mathbf{P}_0$, because of $\mathbf{k}$ and $\Delta\theta$, after a tracking period, the origin of the coordinates will be in $\mathbf{P}_1$ rather than $\mathbf{P}_0$. Next tracking period will be similar, the origin of the coordinates will be jump to $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\cdots$, $\mathbf{P}_n$, if the tracking process is convergent, the origin of the coordinates will converge to $\mathbf{P}_0$ at last, and the slot of the moving origin is just like a helix as Fig.4 shows.

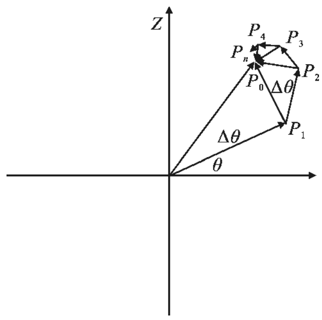

Fig.4 Slot of the origin of coordinates while tracking

Suppose the target location is ρ_0, the target angle is θ which has a bias $\Delta\theta$, and the tracking coefficient is $\mathbf{k}$, which makes an equivalent effect that the origin of the coordinates will be drive to $\mathbf{k}\rho_0$. After the first tracking period, the new origin of the coordinates will be in $\mathbf{P}_1$, as is shown in Fig.4. The three points of $\mathbf{OP}_0\mathbf{P}_1$ form a triangle, $|\mathbf{OP}_0|=\rho$, $|\mathbf{OP}_1|=\mathbf{k}\rho$, $\angle\mathbf{P}_0\mathbf{OP}_1=\Delta\theta$, $|\mathbf{P}_1\mathbf{P}_0|=\rho_1$ and ρ_1 will be the new target distance. According to the law of cosine:

$$\rho_1^2=\rho_0^2+\mathbf{k}^2\rho_0^2-2\rho_0\mathbf{k}\rho_0\cos(\Delta\theta)$$

$$\rho_1=\rho_0(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))^{\frac{1}{2}} \quad (4)$$

After another tracking period, the origin of the coordinates will be in $\mathbf{P}_2$, and $\rho_2=|\overline{\mathbf{P}_2\mathbf{P}_0}|$ will be the new target distance, and by using the law of cosine:

$$\rho_2=\rho_1(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))^{\frac{1}{2}}=\rho_0(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))^{\frac{2}{2}} \quad (5)$$

After $\mathbf{n}$ times tracking period, we can get:

$$\rho_n=\rho_0(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))^{\frac{n}{2}} \quad (6)$$

It is obvious that ρ_n is a power series, if we want to see the convergence property of any series, the two facets must be discussed: the convergent conditions and the convergent step.

Firstly, the convergent conditions of ρ_n. According to the series theory, if the power series is convergent, then every step of the series must be less than 1, that is to say $1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta)$ is less than 1, furthermore, another condition must take into consideration that is $\cos(\Delta\theta)>\frac{\mathbf{k}}{2}$, because the value of cosine function will never be larger than 1.

Secondly, the convergent step of ρ_n. Theoretically, the convergent power series must be approach to zero when the step becomes infinite, but in most applications, the convergent step will not be infinite because of the actual needs. In this application, the tracking precision of the gyro is 0.1% of the target location, so the minimum convergent step will be:

$$0.01\rho_0=\rho_0(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))^{\frac{n}{2}}$$

$$\mathbf{n}=2*\log_{1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta)}{}^{0.001}=2*\frac{\ln 0.001}{\ln(1+\mathbf{k}^2-2\mathbf{k}\cos(\Delta\theta))}\quad(7)$$

Assume the tracking coefficient $\mathbf{k}=(0.5,1.5)$, the angle bias $\Delta\theta=(-\frac{\pi}{18},\frac{\pi}{18})$, by using Matlab, the relationship of $\mathbf{k}$, $\Delta\theta$ and the convergent steps, is shown in Fig.5. If $\mathbf{k}$ is close to 1 and $\Delta\theta$ is close to 0, the convergent step will be close to 1, and with the deviation of the two parameters, the convergent step will be getting larger and larger.

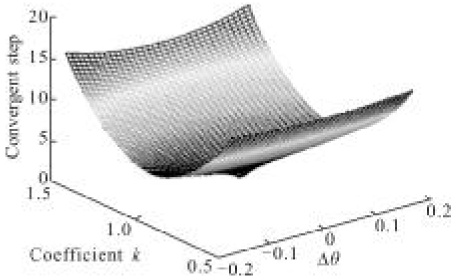

Fig.5 Relationship of convergent steps, $\mathbf{k}$ and $\Delta\theta$

3 Conclusions

The tracking of the reticle seeker is driven by the period mean torque which is generated by the tracking signal. Through mathematical analyses, we can get the three conclusions. Firstly, the convergent property of the tracking procession is affect by both the tracking coefficient $\mathbf{k}$ and the phase bias $\Delta\theta$ of the tracking signal. Secondly, the two parameters must meet the convergent condition that are $\cos(\Delta\theta)>\frac{\mathbf{k}}{2}$ and $\mathbf{k}<2$. Lastly, the convergent step is getting larger if $\mathbf{k}$ deviates from 1 or $\Delta\theta$ deviates from 0 or both. These conclusions are important for the designing the convergent tracking algorithm of the reticle seeker.

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