# Passive ranging based on virtual circle from three matched points 

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#### Abstract

It is critical to extract the linear rotational invariant of an imaged target in passive ranging from the images，and $\mathrm{it}^{\prime} \mathrm{s}$ more difficult to extract the parameters from a general non－cooperative than from a cooperative ones．A method was presented to construct a virtual circle as the inherent rotation invariance of a circular target．The proposed virtual circle was the circumcircle of equilateral by triangles extended from three matched points in adjacent frames in the image sequence．It is demonstrated by the simulations that the probability density function curve of the proposed virtual circle has tighter error distribution than that of a few other methods，and further studies indicate that the diameter of this virtual circle is also a preferable depth－related line segment feature．The line segment feature is used for the target distance estimation and it displays superior performance．It is characterized by its simple for the distance estimation using line segment features，and the concept of virtual circle increases the flexibility in practice．Because as few as three matched points are the least points in target tracking based image feature，so it is attractive for passive ranging to non－cooperative targets．The method is valid under the condition of the inclination angle of target relative to camera increased or decreased from－ 10 degree to 10 degree between adjacent sampling times．


Key words：machine vision；rotation－invariant；pattern recognition；target tracking；range finding
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## 基于三点虚拟圆的被动测距

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#### Abstract

摘 要：在被动定位中，从图像中提取目标的线性旋转不变特征很重要，而非合作目标特征提取要比合作目标困难得多。借鉴圆形目标固有的旋转不变特性，提出了一种虚拟圆方法。该虚拟圆为图像序列相邻帧内三个匹配点经等边三角形向外延拓生成的三个点的外接圆。模拟结果表明，此虚拟圆较其他途径构成的虚拟圆有尖锐的误差分布概率密度函数，进一步研究表明该虚拟圆的直径是一个相当不错的距离相关线段特征。线段特征用于距离估计的特点是简单，虚拟圆概念则增加了它在应用中的灵活性。因为三个匹配点是基于目标特征跟踪的最少匹配点数，故该方法在非合作目标被动定位中具有吸引力。该方法对相邻帧成像时目标相对相机偏斜变化不超过 $\pm 10^{\circ}$ 有效。


关键词：机器视觉；旋转不变；模式识别；目标跟踪；距离测量

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## 0 Introduction

A circular target is inherently rotation-invariant ${ }^{[1]}$ and its image is a circle or an ellipse from any direction. The diameter of imaged circle or the length of the long axis of the imaged ellipse is inversely proportional to the distance from the camera, therefore the diameter or the long axis is the distance-related line feature.

In general, targets are non-cooperative and they don't have round contours or circular marks. As a result the circular rotation-invariance couldn' $t$ be directly utilized, however virtual circles, which still possesses rotation-invariance, could be constructed with a few matched points from adjacent frames of the image sequence. These matched points might be the scale-invariant feature transform(SIFT) key points ${ }^{[2]}$, Harris corners ${ }^{[3]}$, or the speeded up robust feature (SURF) points ${ }^{[4]}$. The concept of virtual circles result from conceptual evolution of depth-related line segment feature ${ }^{[5]}$.

The key of using virtual circle for passive ranging is the construction of the virtual circle. In this paper, a new virtual circle is proposed. It is showed by the simulations that the probability density function curves of the proposed virtual circle has tighter error distribution. The concept of virtual circle is shown in Fig.1.


Fig. 1 Virtual circles
In Fig.l, matched points $A, B, C$ and $A^{\prime}, B^{\prime}, C^{\prime}$ are on the circle $A B C$ or ellipse $A^{\prime} B^{\prime} C^{\prime}$ respectively. Both circle $A B C$ and ellipse $A^{\prime} B^{\prime} C^{\prime}$ is virtual circles.

Because it needs at least 5 points to fit out an ellipse, so there needs 5 points on a circle or on an ellipse to build a virtual circle. This requirement limits its applications. As a result it is more attractive in a real world to study virtual circle based on fewer matched points. In this paper, virtual circles which are less affected by the imaged target swinging with the camera' s focal plane are determined. This virtual circle needs only 3 points assuming that the circle still remained a circle between adjacent frames, as the circle $A B C$ and circle $A^{\prime} B^{\prime} C^{\prime}$ in Fig.1.

## 1 Determination of virtual circles

According to machine vision theory, if imaging distance is much larger than the target size, the imaging process could be modeled by weak perspective projection with a fairly good approximation ${ }^{[6]}$. As shown in Fig. $2^{[7]}$, weak perspective projection is a scaled orthographic projection. First the object is projected onto the image plane by a set of parallel rays parallel to the optical axis $O Z$; then the image of the object is scaled. Weak perspective projection points on the object depth can be approximated as of the same depth value $z_{0}$.

As shown in Fig.2, $\theta$ is the angle between the target and focal plane. The change in $\theta$ between adjacent frames is the swinging angle of the target and it is generally a small one for a 25 frame/s continuous shooting target ${ }^{[8]}$, it is less than $10^{\circ}$ for example. Through this reasonable supposition, $\Delta \theta \in\left[-10^{\circ}, 10^{\circ}\right]$ is uniformly distributed in our simulation. Here, $\Delta \theta$ is used for representing the change in $\theta$.


Fig. 2 Weak perspective mode

Suppose there have been three pairs of matching points $P_{1}-P_{1}{ }^{\prime}, P_{2}-P_{2}{ }^{\prime}$ and $P_{3}-P_{3}{ }^{\prime}$ obtained by image processing，which are the bases for building virtual circles．A few different circle could be constructed with these three points $P_{1}, P_{2}, P_{3}$（or $P_{1}{ }^{\prime}, P_{2}{ }^{\prime}, P_{3}{ }^{\prime}$ ）．Such as

Method－1：circumcircle of triangle $P_{1} P_{2} P_{3}$ ．This method has been used in Ref．［5］before；

Method－2：incircle of triangle $P_{1} P_{2} P_{3}$ ；
Method－3：Euler＇s circle of triangle $P_{1} P_{2} P_{3}$ ．As shown in Fig． $3{ }^{[9]}$ ，the name honors Euler for the first discover；


Fig． 3 Euler＇s circle
Method－4：let $A, B, C$ be the escenters of $\Delta P_{1} P_{2} P_{3}$ ，we get escenters based circumcircle（ $E B C$ ）；

Method－5：a circumcircle of equilateral triangle extension（CETE）is shown in Fig．4．The points $A, B$ ， $C$ are the vertexes of equilateral triangle $P_{1} A P_{2}, P_{2} B P_{3}$ ， and $P_{3} C P_{1}$ respectively．


Fig． 4 Circumcircle of equilateral triangle extension
The virtual circle diameters in $n$ and $n+1$ frame are denoted by $D_{n}$ and $D_{n+1}$ ，so the error caused by the target swinging with the camera focal plane is

$$
\begin{equation*}
\text { Error }=\left(D_{n+1}-D_{n}\right) / D_{n} \tag{1}
\end{equation*}
$$

The probability density distribution curves of the error defined in（1）from the 5 virtual circles are drawn in Fig． 5 after 120000 times of Mont Carlo run
for each method under $\Delta \theta$ uniform distribution from $-10^{\circ}$ to $+10^{\circ}$ ．

The main peaks for each method are at 0.0001 ， 0.02 and 0.2 ．As a matter of fact the error $=0.0001$ can be considered as error $=0$ ．Therefore from Fig． 5 it is clear that method -5 is the best one of those 5 methods which has the highest main peak value and lower second peak value，and method -2 ，method -4 are better than method－1．


Fig． 5 Error probability density distribution curves for each method with local truncation error of 0.5 pixel

According to mathematical expectations result from curves in Fig．5，the best method is method－5，next is method－4．

Because of the wide applications of the SIFT key points in imaging tracking，it is necessary to probe the effects of these virtual circles made from SIFT key points．In the simulation，three points $P_{1}, P_{2}, P_{3}$（or $\left.P_{1}{ }^{\prime}, P_{2}{ }^{\prime}, P_{3}{ }^{\prime}\right)$ are used to build the circumcircle，EBC or CETC，and the diameters are chosen as the line segment features respectively．Each key point has a corresponding vector，which with a random direction and a module value belong to $[60,255]$ in uniform distribution．The error distribution of each virtual circle diameter is displayed in Fig．6．


Fig． 6 Error distributions of each line segment feature

It shows that the virtual circle diameter produced by method-5 has the best rotation invariability.

## 2 Experiment

By substitute the virtual circle diameter as the line segment feature described in Ref.[10], an indoor experiment with reduced model was performed,showed in Fig.7. Figure 8 provides a set of image sequence in which the target's pose changes obviously.


Fig. 7 Experiment with reduced model indoor


Fig. 8 Target sequence one for experiment, (1), (2) et al, is frame numbers

## 3 Conclusion

The concept of suboptimal rotational invariant circle based on three matching points is proposed, five kinds of virtual circles constructed from three matching points are discussed. Simulation yields the error distribution curves of each virtual circle and their expected errors via local truncation error of 0.5 pixels. According to the least squared error (LSE) criteria, CETC and EBC are better than the other three virtual circles. Next, taken emphasis upon the
problem of selection a good line feature from these virtual circles, which are made from three matched SIFT key points. It has been found by our work that the virtual circle diameter of method -5 is one of the best line features. Finally, the line feature was applied to a noncooperative target passive ranging and it demonstrated that the ranging error reached the minimum when the CETC diameter was chosen as the target' s rotationinvariant line segment feature. In conclusion, the proposed method is appropriate for passive ranging of non-cooperative targets provided that the inclination angle of target relative to camera change between adjacent frame in the range of $[-10,10]$ degree.

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