# Accurate frequency estimator for optical coherent M–PSK system based on FFT and multiple signal classification algorithm

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**Abstract:** For optical coherent M –ary phase-shift-keying (M –PSK) system, the frequency offset algorithm based on differential phase or FFT maximization which was widely used is difficult to achieve MHz estimation error when the data length is short, which is difficult for the following carrier phase estimation to recover the data. To meet the needs of high accuracy and real-time performance for frequency offset estimation in the M–PSK system, a frequency estimator based on fast Fourier transform and multiple signal classification (MUSIC) was proposed and investigated. For the first time, MUSIC algorithm was used in this area. The proposed algorithm is accurate especially when the data length is short. The principle and flowchart were proposed to illustrate the algorithm. Numerical simulations of 20–Gbaud QPSK coherent systems were carried out to demonstrate this algorithm.

Key words: coherent detection; fast Fourier transform (FFT); frequency offset estimation;

M-ary phase-shift-keying (M-PSK); multiple signal classification (MUSIC);

optical communication

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# 基于 FFT 和多重信号分类算法的高精度相干光相移键控信号 频率偏移估计算法研究

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摘 要:针对相移键控(MPSK)相干光通信系统中的频率偏移,最常用的两种算法是基于差分相位或 者 FFT 最大值的算法,但是当数据长度较短时,两种算法均很难实现 MHz 的估计误差,这将使得后 续载波相位恢复估计很难恢复原始数据。为满足 MPSK 系统中频率偏移估计算法高精度和实时性的 要求,首次将多重信号分类算法引入该问题,提出一种基于快速傅里叶变换和多重信号分类的频率估 计算法,该算法在数据较短时精度很高。利用基本原理和流程图对算法加以说明,并进行了 20-GBaud/

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sQPSK 相干光系统仿真实验验证算法切实可行。

关键词:相干探测; 快速傅里叶变换; 频率偏移估计; 相移键控; 多重信号分类; 光通信

### **0** Introduction

Coherent detection using digital signal processing (DSP) techniques for free-running local oscillator (LO) lasers has been proposed since 1991<sup>[1]</sup>. But the invention of erbium-doped fiber amplifier (EDFA) makes the intensity modulation/direct detection(IM/DD) dominant in optical communication system. In recent years, coherent detection combined with DSP has renewed interest not only because it can achieve higher spectral efficiency which is suitable for large capacity and high speed communication system, but also enables compensation of transmission impairments such as chromatic dispersion (CD), polarization mode dispersion (PMD), phase noise and so on in electronic domain<sup>[2–3]</sup>.

In DSP-based phase estimations (PEs), feedforward approach is preferred. But in this approach, due to the frequency offset between the transmit laser and local oscillator (LO), the received data constellation for the phase-modulation transport system would suffer a huge degradation and result in a degradation in bit-error-rate(BER). In previous literatures, some frequency offset estimators for the optical coherent M-ary phase-shift-keying (M-PSK) system have been presented<sup>[4-7]</sup>. The widely used frequency offset estimators for optical communications are differential phase based method and FFT maximization based method<sup>[8-9]</sup>. But differential phase based method cannot achieve high accuracy. The FFT algorithm can improve the performance when the FFT point is large, but it is still difficult to achieve high accuracy under the influence of spectral leakage and picket-fence effect. What is more, the both two estimators cannot achieve several MHz estimation error when the data length is short. This would significantly affect the performance of the system and it is difficult for the following carrier phase estimation to recover the data.

The multiple signal classification (MUSIC) algorithm<sup>[10]</sup> is based on model of harmonic signals and has been widely used in array signal processing to estimate the Direction of Arrival (DOA) since it can estimate frequency to infinite precision. But the algorithm has to search spectral peak in the full range frequency domain and results in a large amount of calculation.

In this paper, MUSIC algorithm combined with FFT is proposed to estimate the frequency offset between signal and local oscillator for optical coherent M –PSK system to achieve high accuracy and real-time performance. First *M*th operation is used to erase the modulation phase. Then FFT is used to search the spectral peak and estimate the frequency offset coarsely. At last interpolation and MUSIC are used around the spectral peak to achieve the accurate estimation. Simulations for 20 –Gbaud QPSK are employed to demonstrate this algorithm. The results show that the range of the frequency offset estimation can cover (–2.5 GHz, +2.5 GHz); the average and maximum estimation errors are well below 0.1 MHz and 0.8 MHz, respectively.

### **1** Principle

The *n*th received symbol of coherent M –PSK signal can be described  $as^{[7]}$ 

$$S_n = I_n + jQ_n = \exp[j(\theta_n + \varphi_n + 2\pi\Delta f nT_s)] + N_n$$
(1)

Where  $\theta$  is modulated data phase, and  $\varphi$  is the laser phase noise. The phase difference between two adjacent symbols( $\varphi_n - \varphi_{n-1}$ ) follows Gaussian distribution. Frequency offset between the transmit laser frequency  $f_t$  and the LO frequency  $f_{LO}$  is denoted as  $\Delta f = f_t - f_{LO}$ . Symbol duration is represented as  $T_s$ , and  $N_n$  is amplified spontaneous emission (ASE) noise.

It is obvious that the modulated data phase would be erased after the Mth operation and the signal can be written as:

$$S_n^M = \exp(j2\pi M\Delta f n T_s + M\theta) + N_n'$$
(2)

Performing FFT on formula(2) and searching the maximum in the discrete spectrum, there is a relationship between the peak frequency and the frequency offset<sup>[9,11]</sup>. As is shown in Fig.1, it is the spectrum of the 20 -Gbaud QPSK signal' s 4th power with 1 GHz frequency offset in the simulation. We can see that the peak appears at 4 GHz while the frequency offset is 1 GHz, and the frequency offset can be obtained through the division of the peak frequency by 4.



Fig.1 FFT on the 4th power of the signal

For (N+1) signal, we assume that:

$$e_i = [1, e^{jwi}, \cdots, e^{jwNi}]^{\mathrm{T}}$$
 (3)

$$e_i = [1, e^{jwi}, \cdots, e^{jwNi}] \quad i = 1, 2, \cdots, p$$
 (4)

Where p is the number of sinusoid contained in the signal.

We can get the correlation matrix as follows:

$$R_{y}(\tau) = \sum_{i=1}^{p} e_{i}e_{i}^{H} + \sigma^{2}I$$
(5)

and the singular value decomposition (SVD) of  $R_v(\tau)$  is:

$$R_{v}(\tau) = VSU^{H} \tag{6}$$

Where *V*, *U* are the unitary matrix composed of left and right singular vector of  $R_y(\tau)$ , respectively. *S*= diag( $\alpha_1, \dots, \alpha_N$ ), and  $\alpha$  is the singular value of  $R_y(\tau)$ . Then we set:

$$R = E(R_{y}R_{y}^{H}) = HS^{2}V^{H}$$
(7)

From (7), we can see that the singular vector V of  $R_y(\tau)$  is also the eigenvector of R. So we can get the signal subspace  $V_s$  and noise subspace  $V_n$  from the SVD of  $R_y(\tau)$ . According to the principle of MUSIC, spatial spectra can be constructed in formular(8)

$$P(w) = 1/(\alpha^{H}(w)V_{n}V_{n}^{''}\alpha(w))$$
(8)

When searching the spectral peak of pseudospectral, we can get the signal frequency accurately<sup>[12]</sup>. Fig.2 is the block flowchart of the proposed frequency estimator. The frequency estimator is shown in the red dash dot frame(m=100 in our simulations)



Fig.2 Block diagram of the algorithm

As shown in Fig.2, five steps are used to estimate the frequency offset  $\Delta f$ :

(1) Raise the received symbol to *M*th power so that the modulated data phase is erased.

(2) Compare data length with m (m = 100 in simulations). If data length is less than m, only MUSIC is used because MUSIC is accurate and quick enough to estimate the frequency. Otherwise go to step 3.

(3) Perform FFT on the data and find out the frequency of the maximum in the discrete spectrum. After that, frequency offset is obtained through the division of the peak frequency by M and the coarse estimation is accomplished.

(4) Abstract the former and latter two data around the spectral peak and use interpolation to subdivide the spectral. Finally, MUSIC is used to estimate the frequency offset accurately.

(5) Finally output  $S_n^* = S_n \times \exp(-j2\pi\Delta f n T_s)$ 

Even though using interpolation and MUSIC algorithm would increase the computing time and complexity slightly, this algorithm is more accurate than the FFT based estimators and much more simple and real-time than MUSIC algorithm. The value of m is depending on the practical system to achieve more accurate or more real-time.

## 2 Simulation results

Simulations for 20 - Gbaud QPSK transmission system are carried out to investigate the performance of the proposed algorithm. The data length is set to be 215 and the laser linewidth is 0.1 MHz. Signal-tonoise ratio (SNR) stands for the ratio of the signal power and ASE noise power in the electrical domain. In simulation, we set SNR to be 12 dB and 1 000 simulations for each frequency offset. In Fig.3 (the max estimation error of differential phase is not shown because it is too large), the max estimation error of our estimator is below 0.8 MHz and the mean estimation error is well below 0.1 MHz through the whole estimation range (-2.5 GHz, 2.5 GHz), which covers the range of most frequency offset<sup>[13]</sup>. It clear that the mean estimation error of our is estimator is below that of FFT and much smaller than that of differential phase.



Fig.3 Absolute values of estimation error under(-2.5 GHz, 2.5 GHz) frequency offsets for 20-Gbaud QPSK

Fig.4 shows mean estimation error versus SNR under different data lengths (same as the sample size of FFT), which is denoted as 2 m. The frequency offset is 1 GHz with 1 000 simulations for each SNR. It is clear that the mean error of our estimator is smaller than that of FFT and much smaller than that of differential phase in the case that the SNR is low and data length is long. The mean estimation error is not sensitive to the SNR when the data length is long owing to the SNR improvement from time domain to discrete spectrum during FFT operation<sup>[14]</sup>. While the data length is 25, the mean estimation error declines quickly since only MUSIC is used if the data length is less than 100. We can conclude that the combination of FFT and MUSIC takes advantages of both algorithms to achieve high accuracy and realtime performance simultaneously. The balance between them depends on practical system.



Fig.4 Mean estimation error versus SNR under different data length using FFT, differential phase and estimator

#### **3** Conclusions

An accurate frequency offset estimation algorithm for optical coherent M-PSK system based on FFT and MUSIC algorithm has been proposed and investigated to achieve high accuracy and real-time performance. The estimator has excellent performance especially under the condition of data length is short with high SNR. The principle and flowchart has been proposed to illustrate the algorithm. Numerical simulations for 20-Gbaud QPSK are also carried out to demonstrate the algorithm and the results show that the mean estimation error is as low as 0.1 MHz through the entire estimation range(-2.5 GHz, 2.5 GHz). Meanwhile, the estimation error satisfies the requirement of the following phase estimation significantly. The algorithm is suitable for all M-PSK system and the estimation range covers(-B/(2\*M), +B/(2\*M)), where B denotes baud rate.

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