Influence of optimization model on parameter extraction in Lorentzian Brillouin scattering spectrum

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Abstract: To select the best optimization model on parameter extraction in Lorentzian Brillouin scattering spectrum and improve the accuracy of the temperature or strain measurement, taking the objective functions based on linear and nonlinear least-squares fits as the study objects, the influence of the optimization model on the accuracy of the parameter extraction was systematically investigated. Brillouin scattering spectrum signals with different signal-to-noise ratios, scanned frequency intervals, g_0, ν_B and $\Delta\nu_B$ were numerically generated. The accuracy and computing time of parameter extraction and temperature/strain measurement of the linear model were compared to that of the nonlinear model. The results reveal that, if the signal is ideal and free from noise, the two models are free of any errors. The errors of the two models will increase with increasing in noise level/scanned frequency interval. However, the error of the nonlinear model is appreciably lower than that of the linear one. In engineering practice the nonlinear model should be selected. The analysis is validated with the experimental Brillouin scattering spectrum signals.

Key words: Brillouin scattering spectrum; parameter extraction; least-squares model; temperature/strain measurement

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Lorentzian 型布里渊频谱特征提取时模型的影响

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摘　要: 为了在提取 Lorentzian 型布里渊频谱特征时选择最佳模型,以线性最小二乘法和非线性最小二乘法构造的目标函数为研究对象,系统地比较目标函数构造对频谱特征提取的影响。数值产生了不同信噪比、扫描频率间隔、g_0、ν_B 和 $\Delta\nu_B$ 的布里渊信号,比较了两种模型得到的频谱特征参数的准确性

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和计算耗时,并用真实布里渊散射信号进行了验证。结果表明:在不含干扰的理想状态下两种模型都能达到非常高的精度,随着噪声含量/扫描频率间隔的增加两种模型的准确性都下降,但线性模型明显大于非线性模型,实际时应采用非线性模型。实验测量的布里渊散射谱信号验证了分析结果的正确性。
关键词: 布里渊散射谱; 参数提取; 最小二乘模型; 温度/应变测量

0 Introduction

Since D. Culverhouse proposed the distributed temperature sensing mechanism based on stimulated Brillouin scattering (SBS) in 1989, Brillouin-based distributed optical fiber sensors have gained significant interest for their ability to monitor temperature and strain in many fields[1-2].

Both the dependences of the Brillouin frequency shift and the intensity of the spontaneous Brillouin signal on strain and temperature have formed the basis of such measurements. Therefore, the measuring range and resolution are mainly limited by the accuracy of the Brillouin frequency shift and power measurement. The Brillouin frequency shift is usually obtained by fitting. When the injection pulse width is larger than the acoustic relaxation time, due to the exponential decay of the acoustic wave in fiber core, the Brillouin gain has a Lorentzian profile[3]. Therefore, the parameter estimation of Brillouin scattering can be realized by the least-squares Lorentzian fit[4]. Currently, there are two optimization models for the least-squares Lorentzian fit[5-6]. After some transformation of the Lorentzian function, it is found that the reciprocal of the Brillouin gain is a polynomial of 2th degree in the scanned frequency and the linear least-squares fit model is used to obtain the parameters[5]. The nonlinear least-squares fit model is used in[6], and the Levenberg-Marquardt algorithm is used to optimize the objective function and obtain the parameters. However, there are some questions about the models. For example, whether the two models are equivalent? What are the advantages and shortcomings of the two models? How about their accuracy? Which model should be selected in various practical situations?

To answer the above questions, the linear and nonlinear models are systematically investigated. The accuracy and computing time of the linear and nonlinear models are compared at different values of signal-to-noise ratio(SNR), scanned frequency interval, g_0, ν_B and $\Delta\nu_B$. The results are useful guides for selecting the suitable model for the parameter extraction in Lorentzian Brillouin scattering spectrum. The accuracy of the extracted parameter, temperature/strain measurement can be effectively improved which is beneficial for high accurate and smart measurement of the temperature/strain.

1 Measurement principle

In the Brillouin-based sensing system, temperature and strain are the main measuring parameters which can be obtained by the Brillouin frequency shift and the Stokes power. The Brillouin frequency shift depends linearly on the local strain and temperature of the fiber, which allows the distributed sensing of the two parameters. The connection among the Brillouin frequency shift ν_B, the increment of temperature T, and the strain ε is given by[7]

$$\nu_B(T,\varepsilon)=\nu_{B0}+C_T\Delta T+C_\varepsilon\Delta\varepsilon \quad (1)$$

where C_T is the temperature coefficient in MHz/℃; ΔT is the changes in the temperature in ℃; ν_{B0} is the reference Brillouin frequency shift in MHz; C_ε is the strain coefficient in MHz/$\mu\varepsilon$; $\Delta\varepsilon$ is the changes in the strain in $\mu\varepsilon$. For an ordinary single-mode fiber with a working wavelength of 1 550 nm, the Brillouin frequency shift coefficient on temperature and strain are 1.10 ± 0.02 MHz/℃ and $0.04\,863\pm0.0\,004$ MHz/$\mu\varepsilon$ respectively[8].

Actually, when the injection pulse width is larger than 10 ns, the Brillouin frequency shift is located at the central frequency of an ideal spectrum that can be modeled by a Lorenzian spectral profile in the frequency domain as:

$$g_B(v)=g_0\frac{(\Delta v_B/2)^2}{(v-v_B)^2+(\Delta v_B/2)^2} \tag{2}$$

where v represents frequency in GHz; v_B represents the Brillouin frequency shift in GHz, and it characterizes the difference between the central frequency of the Brillouin scattering spectrum and the frequency of the incident light. The Brillouin frequency shift of single-mode optical fiber is generally about 11 GHz, when the wavelength of incident light is 1 550 nm. Δv_B represents the 3 dB bandwidth of the Brillouin scattering spectrum and it is relevant to the lifetime of phonon (GHz); g_0 represents the peak value of the Brillouin gain spectrum.

2 Optimization model

From Eq. (1) we discover that the accuracy of temperature/strain measurement can be improved by a decrease of error in v_B. Of course, the above formula is an ordinary one for temperature/strain measurement. In fact, by the simultaneous use of v_B and Δv_B, simultaneous measurement of temperature and strain can be achieved. Therefore, accurate extraction of v_B, Δv_B and g_0 is crucial for accurate measurement of temperature and strain.

2.1 Linear least-squares model

Assume that v_i is the ith value of the scanned frequency, $i=0,1,2,\cdots,N-1$, and g_{Bi} is the Brillouin gain of the ith scanned frequency, $i=0,1,2,\cdots,N-1$. The objective function of least-squares problem of the parameters extraction is defined as:

$$E=\frac{1}{2}\sum_{i=0}^{N-1}e_i^2=\frac{1}{2}\sum_{i=0}^{N-1}(g_B(v_i)-g_{Bi})^2 \tag{3}$$

The above optimization belongs to nonlinear least-squares fit and it is of more complex optimization procedure. We can perform the following conversion:

$$\frac{1}{g_B(v)}=\frac{1}{(\Delta v_B/2)^2g_0}v^2+\frac{-2v_B}{(\Delta v_B/2)^2g_0}{}_Bv+\frac{v_B^2}{(\Delta v_B/2)^2g_0}+\frac{1}{g_0} \tag{4}$$

We will assume that $C_1=\frac{1}{(\Delta v_B/2)^2g_0}$, $C_2=\frac{-2v_B}{(\Delta v_B/2)^2g_0}$, $C_3=\frac{v_B^2}{(\Delta v_B/2)^2g_0}+\frac{1}{g_0}$. Therefore, Eq.(4) becomes

$$\frac{1}{g_B(v)}=C_1v^2+C_2v+C_3 \tag{5}$$

Let us assume that $V=[V_2, V_1, I]$, $V_2=[v_0^2, v_1^2, \cdots, v_{N-1}^2]^T$, $V_1=[v_0, v_1, v_{N-1}]^T$, I is a $N\times1$ column vector with elements of 1, $C=[C_1, C_2, C_3]^T$, and $G=[1/gv_B(v_0), 1/gv_B(v_1), \cdots, 1/gv_B(v_{N-1})]$. Then we can write Eq.(5) as:

$$VC=G \tag{6}$$

The optimization of C in Eq.(6) is a linear least-squares problem[5], the solution is as follows:

$$C=(V^TV)^{-1}V^TG \tag{7}$$

Once C has been obtained, the parameters can be calculated by

$$\begin{cases} g_0=4C_1/(4C_1C_3-C_2^2) \\ v_B=C_2/(-2C_1) \\ \Delta v_B=\sqrt{4C_1C_3-C_2^2}/C_1 \end{cases} \tag{8}$$

2.2 Nonlinear least-squares model

The Levenberg-Marquardt algorithm is particularly suited to resolve the nonlinear least-squares fit problem in Eq.(3). $J_{i,j}=\frac{\partial e_i}{\partial w_j}$, $i=0,1,2,\cdots$, $N-1$, $j=0,1,2$, is an element of the Jacobian matrix J, which is represented as:

$$J_{i1}=\frac{\partial e_i}{\partial g_0}=\frac{\Delta v_B^2}{4(v_i-v_B)^2+\Delta v_B^2} \tag{9}$$

$$J_{i2}=\frac{\partial e_i}{\partial v_B}=\frac{8g_0\Delta v_B^2(v_i-v_B)}{((2v_i-v_B)^2+\Delta v_B^2)^2} \tag{10}$$

$$J_{i3}=\frac{\partial e_i}{\partial v_B}=g_0\frac{4\Delta v_B}{4(v_i-v_B)^2+\Delta v_B^2}-g_0\frac{2\Delta v_B^3}{((2v_i-v_B)^2+\Delta v_B^2)^2} \tag{11}$$

Suppose $e=[e_0, e_1, \cdots, e_{N-1}]^T$ is the error vector, and $W=[g_0, v_B, \Delta v_B]^T=[w_1, w_2, w_3]^T$ is the variable vector. I is a 3×3 unit matrix. Thus the variable update formula can be expressed as:

$$W(k+1)=W(k)-(J(k)^TJ(k)+\lambda I)^{-1}J(k)^Te(k) \tag{12}$$

where k is the number of iterations. The parameter λ is multiplied by 10 whenever a step would result in an increased E. When a step reduces E, λ is divided by 10.

The initial guess of the parameters will influence the convergence performance and convergence rate of the following used Levenberg-Marquardt algorithm. The following approach can get the good initial values to ensure the good performance of the Levenberg-Marquardt algorithm. The estimated value of ν_B is equal to the scanned frequency corresponding to the maximum Brillouin gain spectrum. The estimated value of g_0 is equal to the maximum Brillouin gain in the signal. The estimated value of $\Delta\nu_B$ is set to a random value from a third of the whole scanned frequency span up to the whole scanned frequency span.

3 Comparison of the two models

In the optical fiber sensing system, there are multiple sources of noise originated in the light, optical transmission and data acquisition systems [9]. The accumulation of the noise is manifested as additive white noise in signals acquired from the sensing system[8], which is physical assumed as a Gaussian random noise in Refs.[5] and [10]. To better compare errors in various circumstances, the Gaussian white noise is used in the later section.

3.1 Numerically generated signals

Without loss of generality we numerically generate a Brillouin scattering spectrum signal, and g_0, ν_B and $\Delta\nu_B$ are set to 0.7, 10.8 GHz and 0.08 GHz respectively. In practice, the one-time measured Brillouin scattering spectrum signal has high level of background noise, the SNR is very low. Ensemble averaging is the most common method to suppress the noise because it is simple to implement and very effective in removing noise. But even if after thousands of times averaging, the signal still has a certain level of noise. To simulate the disturbances in the real signals, a Gaussian white noise with zero mean is added to the numerically generated signals and the SNR is 25 dB. Because of randomness of the noisy signals, with regards to any combination of parameters, the signal is generated and calculated 10 000 times. The scanned frequency is in the interval of $\nu_B-\Delta\nu_B$ to $\nu_B+\Delta\nu_B$ and 21 pairs of data are obtained. E_{max}, E_{min}, E_{am} and E_{std} respectively represent the maximum value, minimum value, mean value of amplitude and standard deviation of the errors respectively. Statistics of errors of two models for the numerically generated signal are summarized in Tab.1. Assume that the variation in ν_B results solely from temperature or strain. The errors of temperature or strain measurement are tabulated in Tab.2. With reference to the two models, typical fitted curves and the noisy signal are shown in Fig.1.

Tab.1 Statistics of errors in parameter extraction by two models for numerically generated signals

Model	Parameter	E_{max}	E_{min}	E_{am}	E_{std}
Linear	g_0	5.40×10^{-1}	-9.04×10^{-2}	3.37×10^{-2}	4.37×10^{-2}
	ν_B/GHz	6.04×10^{-3}	-6.19×10^{-3}	1.17×10^{-3}	1.48×10^{-3}
	$\Delta\nu_B$/GHz	1.35×10^{-2}	-3.19×10^{-2}	4.12×10^{-3}	4.97×10^{-3}
Nonlinear	g_0	3.09×10^{-2}	-3.22×10^{-2}	7.04×10^{-3}	8.80×10^{-3}
	ν_B/GHz	1.86×10^{-3}	-1.89×10^{-3}	3.95×10^{-4}	4.94×10^{-4}
	$\Delta\nu_B$/GHz	6.70×10^{-3}	-5.64×10^{-3}	1.27×10^{-3}	1.60×10^{-3}

Tab.2 Statistics of errors in temperature/strain measurement by two models for numerically generated signals

Model	Parameter	E_{max}	E_{min}	E_{am}	E_{std}
Linear	Temperature/℃	5.49	−5.63	1.07	1.34
	Strain/$\mu\varepsilon$	1.24×10^2	-1.27×10^2	2.41×10	3.04×10
Nonlinear	Temperature/℃	1.69	−1.72	3.59×10^{-1}	4.49×10^{-1}
	Strain/$\mu\varepsilon$	3.83×10	-3.89×10	8.12	1.02×10

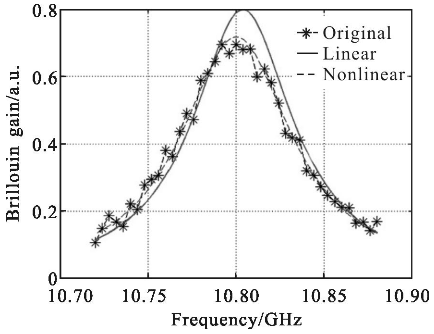

Fig.1 Numerically generated signal and corresponding fitted results

The linear model is relatively faster to compute than the nonlinear one. The mean computing times of the two models are 0.27 ms and 0.63 ms respectively, and the computing time of the former is close to half of that of the latter. The linear model only needs to calculate Eqs.(7)-(8) and no iteration is needed. The nonlinear model must calculate Eqs.(3), (9)-(12) and at the same time, several iterations are essential for obtainment of an accurate solution. From program implement and computing time of view, the linear model is more prone to implement.

From Tab.1 it can be seen that the errors of the linear model is appreciably larger than that of the nonlinear model. To analyze further the error, the sum of the squares of the errors is calculated by Eq.(3), and its maximum and mean values of the linear model in 10 000 runs are 0.94 and 2.05×10^{-2} respectively. For the nonlinear model, the two values are only 2.21×10^{-2} and 1.08×10^{-2} respectively. It is apparent that not only the sum of the squares of the errors, but also the errors in the evaluated parameter value, and the errors of the nonlinear model are appreciably less than that of the linear model. From Tab.1 we discover that the nonlinear model is appreciably more stable than the linear model, especially for g_0.

As seen in Tab.2 the accuracy of the nonlinear model is appreciably higher than that of the linear model. The change in ν_B is proportional to the change in temperature/strain according to Eq.(1). As a result, the data in Tab.2 is proportional to the corresponding data in Tab.1.

From Fig.1 we discover that the curve fitted by the linear model deviates wholly from the given data. As a result, the fitted error in the linear model is of significant magnitude and at the same time the evaluated parameters have significant errors. The fitted curve obtained by the nonlinear model is consistent with the given data. Therefore, the evaluated parameters are more accurate which agrees well with the data in Tab.1 and Tab.2.

Among the errors of the three parameters obtained by the linear model, the error in g_0 is of the most significant magnitude, and its maximum error is 0.77 times of its true value. Even though the maximum Brillouin gain in the signal is simply taken as g_0, relatively speaking, the maximum value, the minimum value, the mean value of amplitude and the standard deviation of the error in g_0 are only 3.09×10^{-2}, -3.22×10^{-2}, 7.04×10^{-3} and 8.80×10^{-3} respectively. The above values are considerably less than the errors of the linear model in Tab.1. That is, the extracted g_0 by the linear model has very significant errors. Among the three parameters obtained by the linear model, $\Delta\nu_B$ is of the highest accuracy. However, its error is still much higher than that of the nonlinear model. The accuracy of ν_B is higher than that of g_0 and lower than that of $\Delta\nu_B$. For the nonlinear model, generally, the errors are negligible. Relatively speaking, regardless of absolute error or relative error, ν_B is of the highest accuracy. Currently ν_B is extensively used to evaluate temperature or strain.

This is due to the fact that although the objective functions in the both two models can be optimized to a value close to the minimum value, different models correspond to different objective functions. This causes the optimal solution to one model is different from that of the other model, even far from the optimal solution to the other model. In the linear model, the objective function is inconsistent with the difference between the Brillouin scattering spectrum and the fitted curve which is responsible for the fact that the obtained optimal solution may differ strongly from the best result. In the nonlinear model, the objective function agrees well with the difference between the Brillouin scattering spectrum and the fitted curve. Therefore, this way has higher accuracy.

If the above Brillouin scattering signals are free from noise, the maximum errors of the two models are both 0.00%. That is to say, if the signal is free from noise and an ideal one, then the extracted parameters are error-free.

To sum up, from an application perspective, the nonlinear model should be selected to extract parameters in Brillouin scattering spectrum, measure temperature and strain. If the unsuitable linear one is selected, several multiples of error will be introduced.

3.2 Real signals

In this section, two real Brillouin scattering spectrum signals are acquired by the N8511 Multi Channel Optical Fiber Strain Sensing System, which is made by ADVANTEST Japan, Inc. An ordinary single-mode fiber with a length of 3.561 km is taken as the test object. The wavelength of the incident light is 1 550 nm, the scanned frequency interval is 10 MHz, and the scanned frequency range is 10.57–10.76 GHz. The numbers of repetitions of signal 1# and 2# are 210 and 211 respectively. The value of the estimated parameter and the computing time by the two models are summarized in Tab.3. The fitted curve and the real one are shown in Fig.2.

Tab.3 Computed result and computing time of two models for two real signals

	Method	g_0	v_B/GHz	Δv_B/GHz	T/ms
1#	Linear	2.15	10.89	0.03	0.24
	Nonlinear	1.04	10.87	0.05	0.97
2#	Linear	0.76	10.88	0.06	0.16
	Nonlinear	1.07	10.87	0.05	0.77

According to Fig.2, regardless of the peak value or waveform, the nonlinear model is in closer agreement on the results than the linear model. For the linear model, the fitted curve always deviates seriously from the real Brillouin scattering spectrum signal in a certain region. Relatively speaking, in spite of the existence of error in the nonlinear model, the fitted curve is entirely consistent with the Brillouin scattering spectrum signal. For the two signals, the sums of the squares of the errors calculated by Eq.(3) in linear model are 1.66 and 3.39×10^{-1} respectively. For the two signals, the sums of the squares of the errors calculated by Eq. (3) in nonlinear model are 2.44×10^{-2} and 1.86×10^{-2} respectively. Clearly, the above error of the nonlinear model is less than a eighteenth of that of the linear model. That is, the nonlinear model is appreciably higher accurate than the linear model which is the same as the analysis in Section 3.1. The computing time of the linear model is about a fourth (fifth) of that of the nonlinear model. The former requires much less computing time than the latter which is consistent with the analysis in Section 3.1.

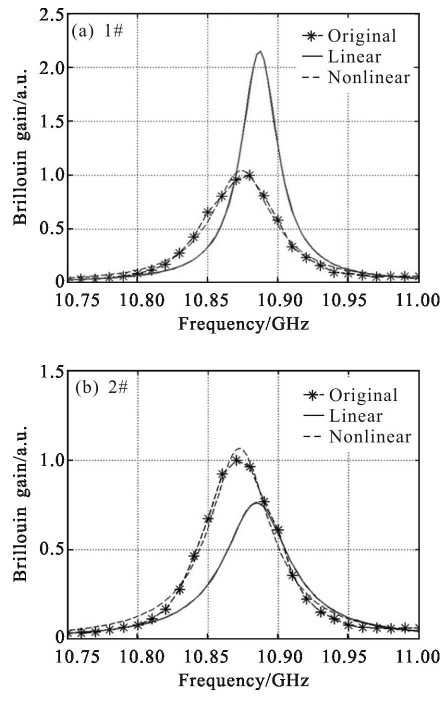

Fig.2 Two real signals and corresponding fitted results

As seen in Fig.2, the obtained v_B by the linear model will deviate significantly from the exact value.

However, the fitted curves agree well with the Brillouin scattering spectrum signals. Therefore, the obtained v_B by the nonlinear model can be taken as an exact value and the value obtained by the linear model has an error. The differences in v_B between the two models are 1.27×10^{-2} GHz and 1.21×10^{-2} GHz respectively. Assume that the variation in v_B results solely from temperature or strain, at the same time the linear model is used, the errors of temperature measurement are 11.55 ℃ and 10.98 ℃, and the errors of strain measurement are 261.32 $\mu\varepsilon$ and 248.27 $\mu\varepsilon$.

4 Conclusion

The influence of the optimization model on the parameter extraction in Lorentzian Brillouin scattering spectrum is systematically investigated. The accuracy and computing time of the linear and nonlinear models are compared at different SNRs, scanned frequency intervals, g_0, v_B and Δv_B. At the same time, the comparison of the two models is conducted for two real Brillouin scattering spectrum signals. The results reveal that:

(1) The linear least-squares model is easy to implement and requires less computing time than the nonlinear one and the computing time of the former is approximately a third of the latter.

(2) The linear model is appreciably more unstable than the nonlinear model, especially for g_0.

(3) If the signal is ideal and free from noise, the extracted parameters by the two models are free of any errors. The errors of the two models will increase with increasing in noise level/scanned frequency interval. However, the error in extracted parameters, measured temperature/strain of the nonlinear model is appreciably lower than that of the linear one. The nonlinear model should be employed which is beneficial for high accurate and smart measurement of the temperature/strain.

(4) For the linear model, the sort of errors in the extracted parameters is $g_0 >> v_B > \Delta v_B$. v_B is of the highest accuracy in the parameters extracted by the nonlinear model.

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