Highly accurate key parameters extraction algorithm for Brillouin scattering spectrum using Voigt profile

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Abstract: The Brillouin scattering spectrum follows Voigt profile. The existing key parameters extraction algorithm for Brillouin scattering spectrum is easy to introduce errors. To ensure high accuracy in the extracted key parameters, the temperature and strain measurement, a key parameters extraction algorithm for Brillouin scattering spectrum using Voigt profile was proposed. The Voigt profile was calculated using the Gauss-Hermite quadrature, the objective function was determined based on the least-squares method and Voigt profile. Besides the initial guesses obtainment method of key parameters was presented. The objective function was optimized using the Levenberg-Marquardt algorithm. Once the objective function was minimized, the key parameters were obtained. Additionally, another algorithm was implemented, in which the initial guesses were set to some random values within a certain range, then the Levenberg-Marquardt algorithm was used to optimize the objective function. A large number of Brillouin scattering spectra with different values of signal-to-noise ratio were numerically generated and measured. The results calculated by the two algorithms reveal that the probability of convergence of the random algorithm fall within a range of 80% to 90%. The proposed algorithm always converges in all cases. The errors by the proposed algorithm are only $1/10^{11}-1/7$ of that by the random algorithm. The computation time by the proposed algorithm is only 1/8-1/3 of that by the random algorithm.

Key words: fiber distributed sensing; Brillouin scattering spectrum; parameters extraction; Voigt profile; temperature/strain measurement

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采用 Voigt 模型的布里渊散射谱关键特征高精度提取方法

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摘 要: 布里渊散射谱满足 Voigt 函数,现有的拟合算法提取得到的特征参数容易存在误差。为了保证布

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里渊谱特征提取时的准确性,从而提高温度和应变测量的准确性,提出了一种采用 Voigt 函数的布里渊散射谱特征参数提取算法。采用了高斯-厄米特积分来计算 Voigt 函数,采用最小二乘拟合方法给出了对应的目标函数,同时给出了初值提取方法,采用 Levenberg-Marquardt 算法来最小化目标函数,一旦目标函数趋于最小化即可获得特征参数。此外还实现另外一种算法,该算法采用随机值方式求解初值,然后采用 Levenberg-Marquardt 算法来优化目标函数。基于数值产生大量不同信噪比的布里渊谱和实测布里渊谱,两种算法的计算结果表明,随机值算法的收敛概率为 80%~90%,文中算法在所有情况下均能收敛。文中算法的计算误差仅为随机值算法的 $1/10^{11}$~1/7,计算耗时仅为随机值算法的 1/8~1/3。

关键词:光纤分布式传感; 布里渊散射谱; 参数提取; Voigt 模型; 温度/应变测量

0 Introduction

Brillouin scattering is extensively used in fiber distributed sensing [1]. The peak value, frequency shift and line width of Brillouin scattering spectrum contain the temperature and strain information of the fiber. The accuracy in the above extracted parameters has an important effect on the accuracy of the temperature and strain measurement.

Generally, the exponential decay nature of the acoustic waves results in a Lorentzian spectral profile [2]. To achieve a high spatial resolution, a narrower pulse is needed. The pulse-pump interaction contributes a Gaussian distribution when the pulse width approaches or narrows below the phonon lifetime[3]. The measured Brillouin scattering spectrum is generally believed to follow Voigt distribution[4-5]. Many papers are reported about key parameters extraction from Lorentzian spectral profile [5-6]. The Lorentzian function belongs to the algebraic equation and it is easily handled. The related study is relatively mature. The Voigt profile is a line profile resulting from the convolution of a Lorentzian profile and a Gaussian profile [7]. However, it is not an algebraic equation. Computation of the profile with high accuracy and low computational effort is not trivial, not to mention key parameters extraction from the Voigt spectral profile. Due to the computational expense of the convolution operation, the Voigt profile is often approximated using a linear combination of a Lorentzian profile and a Gaussian profile which is named as pseudo-Voigt profile[8]. Its computational burden is similar to that of the Lorentzian or Gaussian profile. Therefore, researchers generally fit Brillouin scattering spectrum with a pseudo-Voigt profile [8-10]. However, the pseudo-Voigt profile is different from the Voigt profile. So, these methods introduce error inevitably and the Voigt profile should be considered in the key parameters extraction. There are few papers about this topic. Reference [4] fits the Voigt profile to the measured Brillouin scattering spectrum of a 36 km long-range optical fiber. Nevertheless, most of the details about the key parameters extraction algorithm are not provided. To sum up, there is no effective key parameters extraction algorithm for Brillouin scattering spectrum with Voigt profile and high spatial resolution. This topic needs to be further studied.

To fix the above problem, the objective function is determined according to the least-squares method and the Voigt profile, the initial guesses obtainment method of key parameters is presented, the objective function is optimized using the Levenberg-Marquardt algorithm. Once the objective function is minimized, the key parameters are obtained. Additionally, another algorithm in which the initial guesses are set to some random values, then the Levenberg – Marquardt algorithm is used to optimize the objective function. Key parameters extraction results from a large number of numerically generated and measured Brillouin scattering spectra validate the proposed algorithm.

1 Applicability of the existing algorithms

1.1 Introduction of Voigt profile

Voigt profile[4, 11] can be expressed as:

$$g_B(v)=A\frac{2\ln2}{\pi^{3/2}}\frac{\Delta v_{BL}}{\Delta v_{BG}^2}\int_{-\infty}^{+\infty}\frac{e^{-x^2}}{\left[\sqrt{\ln2}\frac{\Delta v_{BL}}{\Delta v_{BG}}\right]^2+\left[2\sqrt{\ln2}\frac{v-v_B}{\Delta v_{BG}}-x\right]^2}dx \quad (1)$$

where v is the frequency, with a unit of GHz; v_B is the Brillouin frequency shift, with a unit of GHz, which characterizes the difference between the frequency of the incident light and the central frequency of the Brillouin scattering spectrum; Δv_{BL} and Δv_{BG} respectively are the full width at half maximum (FWHM) bandwidths of the Lorentzian and Gaussian profiles, with a unit of GHz; A is a parameter associated with the amplitude of Brillouin scattering spectrum. It is assumed that Δv_B and g_{BM} respectively represent the FWHM bandwidth and the peak value of Brillouin scattering spectrum.

1.2 Applicability of the existing algorithms

Before development of a new algorithm, we should check the applicability of the existing algorithms. Brillouin scattering spectra are numerically generated according to Eq. (1). Without loss of generality, v_B, A and Δv_{BL} are set to 11.8, 0.2, 0.01 GHz respectively. Δv_{BG} ranges from 0.01 to 0.12 GHz. The existing Lorentzian[4], Gaussian[9] and pseudo–Voigt[12] profiles based key parameters extraction algorithms are used. The relative errors in the extracted g_{BM}, v_B and Δv_B are included in Tab.1.

Tab.1 Relative errors in the key parameters extracted by different algorithms for Voigt–type Brillouin scattering spectra

Algorithm	Parameters	Δv_{BG}/Hz					
		0.01%	0.03%	0.05%	0.07%	0.09%	0.11%
Lorentzian	g_{BM}	3.77%	5.48%	5.77%	5.87%	5.97%	6.2%
	v_B	0	0	0	0	0	0
	Δv_B	−8.22%	−12.3%	−13%	−13.21%	−13.3%	−13.45%
Gaussian	g_{BM}	−4.33%	−1.55%	−0.91%	−0.63%	−0.44%	−0.15%
	v_B	0	0	0	0	0	0
	Δv_B	10.43%	3.64%	2.07%	1.42%	1.07%	0.73%
Pseudo–Voigt	g_{BM}	0.12%	0.04%	0.02%	2.53%	2.6%	2.79%
	v_B	0	0	0	0	0	0
	Δv_B	0	0.01%	−0.04%	−5.69%	−6.08%	−6.49%

From Tab.1 it can be seen that all the existing three algorithms can accurately extract v_B at different values of Δv_{BG}. However, the errors will increase with decreasing signal–to–noise ratio (SNR). There are different errors in the g_{BM} and Δv_B extracted by the three algorithms. The errors in g_{BM} and Δv_B extracted by the Lorentzian and pseudo–Voigt profiles based algorithms increase with increasing Δv_{BG}. Conversely, the errors in g_{BM} and Δv_B extracted by the Gaussian profile based algorithm decrease with increasing Δv_{BG}. The maximum errors in g_{BM} extracted by the above three algorithms are 6.41%, −4.33% and 0.44%, respectively. The maximum errors in Δv_B are −13.62%, 10.43% and −0.37%, respectively. The above results reveal that for the Brillouin distributed sensing (Brillouin scattering spectrum is generally believed to follow Voigt distribution), the existing algorithms may introduce significant errors. The pseudo–Voigt profiles based key parameters extraction algorithm may introduce more signficant error for noisy spectra. So, the key parameters extraction algorithm needs to be further studied. This is the core content of our work.

2 Key parameters extraction algorithm for Voigt -type brillouin scattering spectrum

2.1 Objective function

Because Eq. (1) does not have an analytical solution, it must be solved numerically. Gauss-Hermite quadrature [13] is particularly suitable for approximating the value of integrals with e^{-x^2}. That is:

$$\int_{-\infty}^{+\infty} e^{-x^2} f(x)\mathrm{d}x \approx \sum_{m=1}^{M} w_m f(x_m) \tag{2}$$

where M is the number of sample points used. The x_m are the roots of the physicists′ version of the Hermite polynomial $H_M(x)$ with an order of M, (m=1, 2, ..., M), w_m are the associated weights and are given by Eq.(3).

$$w_m = \frac{2^{M-1} M!\sqrt{\pi}}{M^2 [H_{M-1}(x_m)]^2} \tag{3}$$

In consideration of Eq. (2), Eq. (1) can be approximately expressed by:

$$g_B(v) \approx A \frac{2\ln 2}{\pi^{3/2}} \frac{\Delta v_{BL}}{\Delta v_{BG}^2} w_m \frac{1}{\left[\sqrt{\ln 2}\frac{\Delta v_{BL}}{\Delta v_{BG}}\right]^2 + \left[2\sqrt{\ln 2}\frac{v-v_B}{\Delta v_{BG}} - x_m\right]^2} \tag{4}$$

Let v_i and g_{Bi} respectively be the i^{th} scanning frequency and the corresponding Brillouin gain, where i=0, 1, 2, ⋯, N-1, N is the number of frequency scanning. The objective function based on the least-squares method is given by:

$$E = \sum_{i=0}^{N-1} e_i^2 = \sum_{i=0}^{N-1} (g_B(v_i) - g_{Bi})^2 \tag{5}$$

where E is the sum of the squared normal distances between any profile coordinate and the Voigt profile. e_i is the normal distances of the individual coordinate which is defined by:

$$e_i = g_B(v_i) - g_{Bi} = A \frac{2\ln 2}{\pi^{3/2}} \frac{\Delta v_{BL}}{\Delta v_{BG}^2} \sum_{m=1}^{M} w_m \frac{1}{\left[\sqrt{\ln 2}\frac{\Delta v_{BL}}{\Delta v_{BG}}\right]^2 + \left[2\sqrt{\ln 2}\frac{v_i - v_B}{\Delta v_{BG}} - x_m\right]^2} - g_{Bi},$$

$$i = 0, 1, 2, \cdots, N-1 \tag{6}$$

2.2 Optimization method

The above objective function belongs to the nonlinear least-squares problem. The Levenberg-Marquardt algorithm can adaptively adjust from the first-order steepest-descent direction to the second -order Newton direction according to variation of error. It is particularly well suited to solve the above nonlinear least-squares problem. Therefore, the objective function is minimized by the algorithm. The variable update formula can be expressed as:

$$W(l+1) = W(l) - (J(l)^T J(l) + \lambda I)^{-1} J(l)^T e(l) \tag{7}$$

where $e=[e_0, e_1, \cdots, e_{N-1}]^T$ is the error vector, and $W=[W_1, W_2, W_3, W_4]^T=[A, v_B, \Delta v_{BL}, \Delta v_{BG}]^T$ is the variable vector. J is the Jacobian matrix. Because of the limited space, the details on the Jacobian matrix are not presented in the paper. I is a 4×4 unit matrix. l is the number of iterations. Superscript T means transposition. λ is multiplied by 10 whenever a step would result in an increased E. At the same time, the change in the variables is disregarded, and the previous values of the variables are retained. When a step reduces E, λ is divided by 10. The initial value of λ, λ_0 is set to 1. After a large number of trials, the maximum allowable number of iterations L is set to 500.

2.3 Initial guesses obtainment method

The initial guesses of A, v_B, Δv_{BL} and Δv_{BG} will significantly influence the convergence rate and convergence possibility of the objective function. So, we need to find a method to obtain initial guesses with high accuracy and low computational effort.

Assume that the Brillouin gain reaches its peak value if $v=v_P$. Then, the initial guess of v_B can be calculated by Eq.(8).

$$v_B=v_P \tag{8}$$

After a large number of initial attempts, the initial guess of A can be obtained by Eq.(9).

$$A=g_{BM}\Delta v_B \tag{9}$$

The FWHM of the Voigt profile Δv_B can be found from the widths of the associated Gaussian and Lorentzian widths. A good approximation with an accuracy of 0.02% is given by Ref.[14].

$$\Delta v_B\approx 0.534\,6\Delta v_{BL}+\sqrt{0.216\,6\Delta v_{BL}^2+\Delta v_{BG}^2} \tag{10}$$

Since we don't know the values of Δv_{BL} and Δv_{BG} beforehand, let us assume that $\Delta v_{BL}=\Delta v_{BG}$, the initial guesses of Δv_{BL} and Δv_{BG} can be calculated by Eq.(11).

$$\Delta v_{BL}=\Delta v_{BG}=\Delta v_B/1.637\,6 \tag{11}$$

Although the initial guesses obtained in this section have some errors, we can minimize the errors using the optimization algorithm presented in section 2.2.

2.4 Flowchart of the proposed algorithm

The flowchart of the proposed key parameters extraction algorithm from Brillouin scattering spectrum is shown in Fig.1.

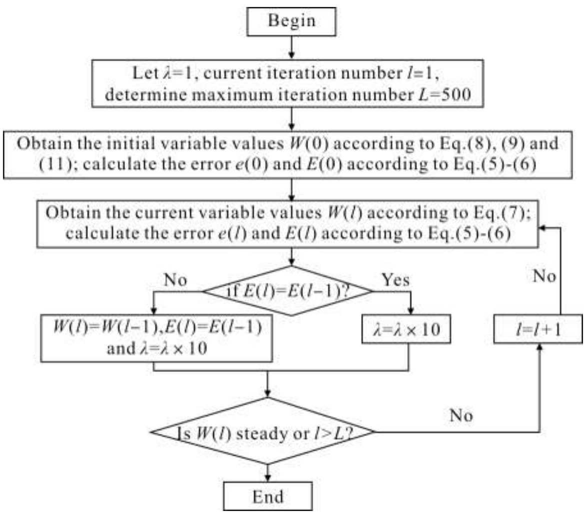


Fig.1 Flowchart of the proposed algorithm

3 Validation

3.1 Numerically generated spectra signals

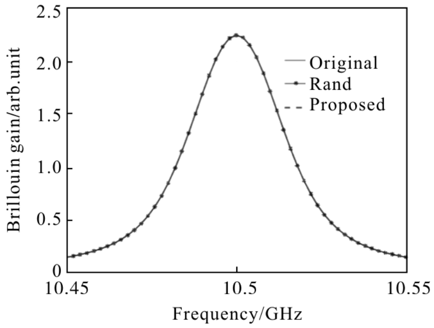
For comparison, we implement another key parameters extraction algorithm in which the initial guesses are set to some random values, then the Levenberg–Marquardt algorithm is used to minimize the objective function. That is, with the exception of the initial guesses obtainment, the random algorithm is the same as the proposed algorithm. In the random algorithm, with the consideration of practical situations, Δv_{BL} and Δv_{BG} are set to random values ranging from 0.01 GHz to 0.15 GHz, and v_B is set to a random value varying from 10 GHz to 13 GHz, and A is set to a random value varying from 0 to 0.3. A large number of noise–free Brillouin scattering spectra and noisy ones with a SNR of 20 dB are numerically generated according to Eq.(1) and the key parameters are set to random values within the same ranges as the random algorithm. After repeated attempts, the number of sample points in the Gauss–Hermite quadrature is set to 100. To validate the proposed algorithm, the above two algorithms are used to extract key parameters from the numerically generated spectra. The statistical errors in the extracted parameters are presented in Tab.2 and at the same time, the computation times are included. $E_{v\text{mean}}$, $E_{\Delta v\text{mean}}$ and $E_{g\text{mean}}$ mean the average values of the error amplitude in the extracted v_B, Δv_B and g_{BM}, respectively. $E_{v\text{std}}$, $E_{\Delta v\text{std}}$ and $E_{g\text{std}}$ mean the standard deviations of the above errors.

As shown in Tab. 2, the mean error amplitude in the key parameters extracted by the random algorithm is significant. For the noise–free spectra, the error in random one is 98–+∞ times larger than that in the proposed one. For the noisy spectra, the error in random one is $7-10^{11}$ times larger than that in the proposed one. The above results

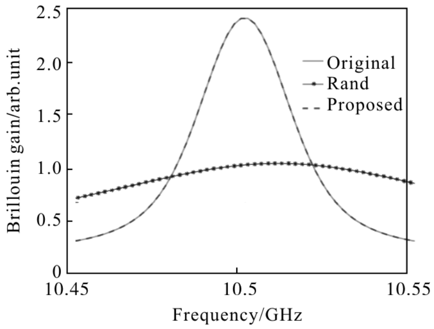
Tab.2 Statistical errors in the key parameters extracted by two algorithms for noise-free and noisy Brillouin scattering spectra

		$E_{\nu mean}$/GHz	$E_{\nu std}$/GHz	$E_{\Delta\nu mean}$/GHz	$E_{\Delta\nu std}$/GHz
Noise-free	Rand	2.49×10^{4}	1.29×10^{5}	4.85×10^{5}	3.91×10^{7}
	Proposed	0	0	7.88×10^{-5}	3.02×10^{-4}
Noisy	Rand	1.10×10^{8}	8.44×10^{8}	6.11×10^{7}	5.23×10^{8}
	Proposed	1.11×10^{-3}	1.48×10^{-3}	3.95×10^{-3}	5.63×10^{-3}
		E_{gmean}	E_{gstd}	T/s	
Noise-free	Rand	2.85×10^{-2}	2.25×10^{-1}	2.65×10^{-1}	
	Proposed	2.92×10^{-4}	1.21×10^{-3}	6.94×10^{-2}	
Noisy	Rand	1.28×10^{-1}	2.9	6.68×10^{-1}	
	Proposed	1.83×10^{-2}	3.58×10^{-2}	2.34×10^{-1}	

reveal that the initial guesses given by Eq.(8)-(9) and (11) are good. Once convergence, both the random algorithm and the proposed one will obtain the optimal solution (see Fig.2(a) and 3(a)) and the divergence cases are also shown schematically in Fig.2(b) and 3(b). The proposed algorithm always converges in all cases. The convergence possibilities of the random algorithm for the noise-free and noisy spectra are 92.28% and 91.57%, respectively.

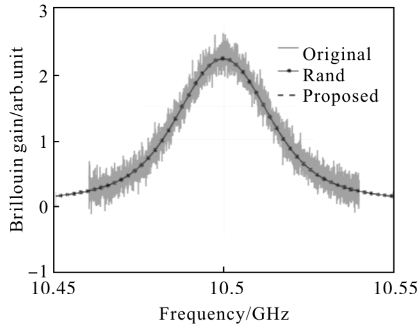


(a) Random algorithm converges

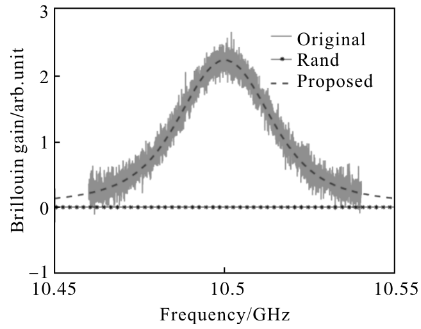


(b) Random algorithm diverges

Fig.2 Curves fitted by two algorithms for noise-free Brillouin scattering spectrum ($\Delta\nu_{BL}$=0.02 GHz, $\Delta\nu_{BG}$=0.02 GHz)



(a) Random algorithm converges



(b) Random algorithm diverges

Fig.3 Curves fitted by two algorithms for noisy Brillouin scattering spectrum ($\Delta\nu_{BL}$=0.02 GHz, $\Delta\nu_{BG}$=0.02 GHz)

The random algorithm not only introduces more considerable errors but also requires more computational effort. The computation times of the random algorithm for the noise-free cases and the noisy ones respectively are 3.82 and 2.85 times that of the proposed one. This is due to the fact that the initial guesses presented by the random one generally deviates very significantly from the optimal solution. Regardless of convergence or not, more numbers of iterations are needed. The above results reveal that even with the help of the effective Levenberg-Marquardt algorithm, the random algorithm will still introduce significant errors and be computationally expensive. The above results validate the proposed algorithm.

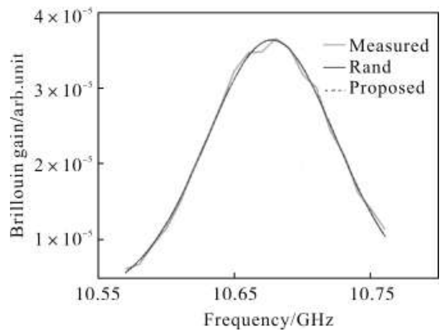
3.2 Measured spectra signals

A Corning LEAF (Large effective area fiber) fiber with a length of 9.534 km is used. The real Brillouin scattering spectrum signal is measured by a Brillouin optical time-domain reflectometer (N8511, ADVANTEST Corporation, Japan). The wavelength of the incident light is 1 550 nm, and the pulse width is 10 ns with a spatial resolution of 1 m. The Brillouin scattering spectra along the fiber with a temperature ranging from 5 °C to 80 °C are measured and a typical one is chosen. To demonstrate the influence caused by the randomness, the random algorithm runs 10 000 times. The extracted key parameters, the fitting error calculated by Eq.(5), the number of iterations and the computation time of the random algorithm are displayed in Tab.3. For comparison, the corresponding results of the proposed algorithm are also summarized in Tab.3. To clearly demonstrate the results, the fitted curve and the measured one in convergence and divergence cases are shown schematically in Fig.4, and the curves fitted by the proposed algorithm are also displayed.

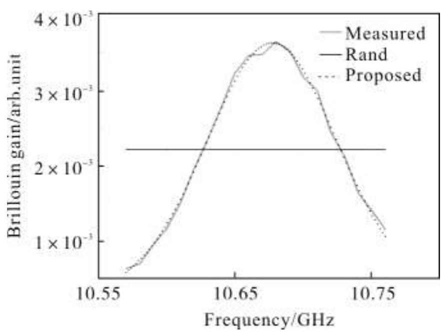
The convergence possibility of the random algorithm is 80.78%. Once divergence, the random algorithm will introduce appreciable errors. The extracted Δv_B and g_{BM} may be not a number or infinity. Therefore, the errors for the convergence and divergence cases are displayed separately. Once convergence, the key parameters extracted by the random algorithm will be the same as these of the proposed algorithm. However, once divergence, the errors introduced by the random algorithm will be considerably larger than those of the proposed algorithm. The above results are quite similar to the results of the numerically generated spectra. That is, the accuracy of the proposed algorithm is much higher than that of the random algorithm. The fitting error of the random algorithm is about 52 times larger than that of the proposed algorithm. The number of iterations and the computation time of the random algorithm are about 13 and 8 times that of the proposed algorithm, respectively.

Tab.3 Results obtained by two algorithms for real Brillouin scattering spectra

	v_B/GHz	Δv_B/GHz	g_{BM}
Rand, convergence	1.07×10	1.20×10^{-1}	3.63×10^{-3}
Rand, divergence	-8.36×10^{10}	NaN	NaN or Inf
Proposed	1.07×10	1.20×10^{-1}	3.63×10^{-3}
	E	Iteration number	T/s
Rand	3.97×10^{-6}	2.40×10^{2}	3.49×10^{-1}
Proposed	7.70×10^{-8}	1.80×10	4.25×10^{-2}



(a) Random algorithm converges



(b) Random algorithm diverges

Fig.4 Curves fitted by the two algorithms, real Brillouin scattering spectrum

4 Conclusion

The aim of the present study is to extract key parameters from Brillouin scattering spectrum with Voigt profile which can be used in Brillouin distributed sensing. The Voigt profile is calculated by use of the Gauss–Hermite quadrature, the objective function is determined based on the least–squares method, the initial guesses obtainment method of key parameters is presented, the objective function is optimized using the Levenberg–Marquardt algorithm. Once the objective function is minimized, the key parameters are obtained. Based on the above investigations, an algorithm to extract the key parameters is proposed and is validated by calculation of numerically generated and real Brillouin scattering spectra. The results reveal that the proposed algorithm always converges in all cases. The errors in the extracted parameters in the proposed algorithm is only $1/10^{11}$–1/7 of that by the random algorithm. The computation time of by the proposed algorithm is only 1/8–1/3 of that by the random algorithm. This work provides support for highly accurate temperature and strain distributed measurement based on Brillouin scattering.

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