Scattering properties of different rarefied random distributed ice crystal particles with different laser wavelengths

Wang Mingjun[1], Yu Jihua[1], Liu Yanxiang[2], Gao Xiangxiang[3], Zhang Huayong[4]

(1. School of Automation and Information Engineering, Xi′an University of Technology, Xi′an 710048, China;
2.CRRC Tangshan Co.,Ltd.,Tangshan 064000, China; 3. North China Vehicle Research Institute, Beijing 100072, China;
4. School of Electronics and Information Engineering, Anhui University, Hefei 230039, China)

Abstract: According to laser statistical scattering properties of ice crystal particles with rarefied random distribution, differential scattering section (DSS) of ice spheres with exponential distribution, lognormal distribution, Gamma distribution changed with scattering angles were calculated numerically and analyzed at 0.65, 1.31 and 1.55 μm, respectively. The results show the change of incident laser wavelength has a certain effect on DSS of ice-crystal layer. And DSS of ice spheres with exponential distribution is the largest, which is several orders of magnitude larger than the other two distributions. So different laser wavelength and size distribution models of ice particles have great influence on laser scattering properties of rarefied random distributed ice crystal particles. The works in this paper are based on the foundation for further study development of the effect of ice-crystals cloud on the laser propagation properties in the ground-air links.

Key words: light scattering properties;　ice crystal particles;　size distribution;
differential scattering section

CLC number: O436　　**Document code:** A　　**DOI**: 10.3788/IRLA201948.0311002

多激光波长在不同稀薄随机分布冰晶粒子层的散射特性

王明军[1]，于记华[1]，刘雁翔[2]，高香香[3]，张华永[4]

(1. 西安理工大学 自动化与信息工程学院，陕西 西安 710048；
2.中车唐山机车车辆有限公司，河北 唐山 064000；3. 中国北方车辆研究所，北京 100072；
4. 安徽大学 电子与信息工程学院，安徽 合肥 230039)

摘　要： 依据稀薄随机分布冰晶粒子的激光散射特性，当球形冰晶粒子分别服从指数、对数正态、Gamma 三种不同分布时，数值计算并分析了 0.65、1.31、1.55 μm 激光入射下不同稀薄随机冰晶粒子层的微分散射截面随散射角的变化关系。结果表明：入射激光波长的改变对冰晶粒子层的微分散射截面有一定的影响；当冰晶粒子服从指数分布时稀薄随机分布冰晶粒子层的微分散射截面最大，要比其他两个分布大几个数量级；不同激光波长和冰粒子的尺度分布对稀薄随机分布冰晶粒子层的激光散射特性有较大影响。文中所做的工作为进一步开展地空链路中冰晶粒子云层对激光传输特性的影响研究奠定基础。

关键词： 光散射特性；　冰晶粒子；　尺度分布；　微分散射截面

收稿日期： 2018-11-10；　**修订日期：** 2018-12-20

基金项目： 国家自然科学基金(61771385，61377080，60977054)；陕西省重点产业创新项目(2017ZDCXL-GY-06-01)

作者简介： 王明军(1979-)，男，教授，博士生导师，博士，主要从事光散射特性理论建模及激光雷达成像技术方面的研究。Email:wmjxd@aliyun.com

通讯作者： 于记华(1992-)，男，硕士生，主要从事水云和冰云的散射与辐射传输特性方面的研究。Email:yjhyujihua@qq.com

0 Introduction

According to the definition of the World Meteorological Organization (WMO), clouds with a base height above 6 km are designated as high clouds, including cirrus (Ci), cirrostratus (Cs) and cirrocumulus (Cc). Because of these clouds with high altitude and low temperature, which are composed of ice particles, and it is also known as ice cloud[1]. Ice cloud is one of the discrete random media, it is great significant to study the propagation and scattering properties of laser through ice cloud for atmospheric detection, communication and remote sensing[2-5]. Therefore, more and more scholars have been focused on the scattering properties of ice particles. It is assumed that the dimensional distribution of ice particle consistented with the Γ function in cirrus clouds, Zijun Liao et al. calculated the average extinction efficiency, absorption efficiency, single scattering albedo and asymmetry factor of cirrus cloud in the visible light spectral region [6]. Combining the discrete ordinate method (DISORT) and single-scattering properties of different shaped ice particles, Yanjie Zhao et al. have studied the scattering and radiative properties of cirrus clouds made of solid hexagonal prismy shape ice particles at 1.315 μm[7]. Based on a combination of the Amsterdam discrete dipole approximation (ADDA), the T-matrix method, and the improved geometric optics method (IGOM), Yang et al. have systematically studied the scattering, absorption, and polarization properties of common ice particles of ice clouds in the spectral range from 0.2 to 100 μm, and developed the single-scattering database of ice particles[8]. Most scholars have studied the light scattering properties of static ice particles[3-10]. However, ice particles in the atmosphere are usually accompanied by certain motion due to the effects of gravity and wind[1]. With the rapid development of aerospace, meteorological technology in recent years, research on laser scattering properties of ice particles in motion state have became more and more important. Therefore, it is of great importance and extensive application value to study the scattering properties of moving particles from a theoretical perspective.

On the basis before works of our group[11], laser statistical scattering properties of ice crystal particles with rarefied random distribution under moving condition are further improved, and scattering properties of three different rarefied random distributed ice crystal particles with different laser wavelengths are numerically calculated and compared in this paper. Our works are provided theoretical models for further study laser propagation through ice clouds.

1 Laser scattering properties of ice particles with rarefied random distribution

According to Mie theory, the scattering fields of a static ice sphere can be given[12]:

$$E_r=0,\ E_\phi=-\frac{i\exp(ikr)}{kr}S_1(\theta)\sin\phi,$$

$$E_\theta=\frac{i\exp(ikr)}{kr}S_2(\theta)\cos\phi \tag{1}$$

with the scattering amplitude $S_1(\theta)$, $S_2(\theta)$:

$$S_1(\theta)=\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}[a_n\pi_n(\cos\theta)+b_n\tau_n(\cos\theta)] \tag{2}$$

$$S_2(\theta)=\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}[a_n\tau_n(\cos\theta)+b_n\pi_n(\cos\theta)] \tag{3}$$

Where Mie coefficient a_n, b_n and π_n, τ_n only related to scattering angle[1,12]. When the particle is moving, the coordinate system is built to analyze the motion of ice sphere as shown in Fig.1. Σ' is static coordinate system, Σ is moving coordinate system. Based on the Maxwell equation Eq.(4), and when $\vec{v}=c$, $[\vec{E}']$ will be also satisfied with Eq.(5).

$$\begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix}=\frac{i}{\omega}\times\begin{bmatrix} \frac{\vec{e}_r}{r^2\sin\theta} & \frac{\vec{e}_\theta}{r\sin\theta} & \frac{\vec{e}_\phi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ E_r & rE_\theta & r\sin\theta E_\phi \end{bmatrix} \tag{4}$$

$$[\vec{E}']=[\vec{E}]+[\vec{v}]\times[\vec{B}] \tag{5}$$

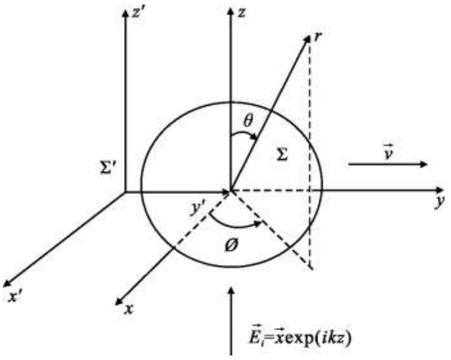

Fig.1 Coordinate system for calculating differential scattering section of particle

From Eqs. (1) –(5), it can be deduced that the components of the scattering field $[\vec{E}']$ of moving ice sphere are:

$$\vec{E}_r'=\frac{\exp(ikr)}{\omega kr^2}\cdot S_1(\theta)\sin\phi v_{\phi_1}-\frac{\exp(ikr)}{\omega kr^2}\cdot S_2(\theta)\cos\phi v_{\theta_1}\tag{6}$$

$$\begin{aligned}\vec{E}_\theta'=&\frac{i\exp(ikr)}{kr}S_2(\theta)\cos\phi+\frac{v_{\phi_1}\exp(ikr)}{\omega kr^2\sin\theta}\{\cos\theta S_1(\theta)-\\&\sin^2\theta\cdot[\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}(a_n\pi_n'(\cos\theta)+b_n\tau_n'(\cos\theta))]-\\&S_2(\theta)\}\sin\phi+\frac{v_r\exp(ikr)}{\omega kr^2}S_2(\theta)\cos\phi\end{aligned}\tag{7}$$

$$\begin{aligned}E_\phi'=&-\frac{i\exp(ikr)}{kr}S_1(\theta)\sin\phi-\frac{v_r\exp(ikr)}{\omega kr^2}\cdot S_1(\theta)\sin\phi-\\&\frac{v_{\theta_1}\exp(ikr)}{\omega kr^2\sin\theta}\cdot\{\cos\theta S_1(\theta)-\sin^2\theta[\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}\\&(a_n\pi_n'(\cos\theta)+b_n\tau_n'(\cos\theta))]-S_2(\theta)\}\sin\phi\end{aligned}\tag{8}$$

Where $\pi_n'=\mathrm{d}\pi_n/\mathrm{d}\theta$, $\tau_n'=\mathrm{d}\tau_n/\mathrm{d}\theta$. According to $f'(\vec{e}_r,\vec{r})=\frac{i}{k}(E_r'\vec{e}_r+E_\theta'\vec{e}_\theta+E_\phi'\vec{e}_\phi)$, the expression of DSS of spherical ice particle is given[12]:

$$\sigma_d(\hat{o},\hat{i})=\lim_{R\to\infty}[(R^2S_S)/S_i]=|\vec{f}(\hat{o},\hat{i})|^2\tag{9}$$

In order to facilitate numerical calculation, two special cases are discussed in this paper:

(1) When the velocity $\vec{v}$ is zero, that is, the ice particle is static, and the components of the velocity $\vec{v}$ are zero. Substituting them into Eqs.(6–8), it can be proved $\vec{E}'=\vec{E}$ that is consistent with Eq.(5). Now, the DSS of static ice sphere can be derived from Eq.(9):

$$\sigma_d(\hat{o},\hat{i})=|\vec{f}'(\hat{o},\hat{i})|^2=\frac{S_2(\theta)^2\cos^2\phi+S_1(\theta)^2\sin^2\phi}{k^4r^2}\tag{10}$$

(2) When particle is moving along the $\vec{y}$ axis at the speed $\vec{v}$, then $v_y=v_r=\sin\theta_1\sin\phi_1$. Now $\sin\theta_1=1$, $\sin\phi_1=1$, $v_y=v_r$, $\cos\theta_1=0$, $\cos\phi_1=0$, $v_{\theta_1}=0$, $v_{\phi_1}=0$, substituting them into Eqs.(6–8), the DSS of moving ice sphere can be derived

$$\sigma_d(\hat{o},\hat{i})=|\vec{f}'(\hat{o},\hat{i})|^2=\frac{S_2(\theta)^2\cos^2\phi+S_1(\theta)^2\sin^2\phi}{k^4r^2}\left(1+\frac{v_r^2}{\omega^2r^2}\right)\tag{11}$$

When the ice particles in the random medium are very tenuous, the laser on the transmission path are scattered only by a small number of particles. So the single scattering is considered only and multiple scattering can be neglected that widely used in meteorology, ocean and interstellar dust[11]. The ice particle size spectrum may be represented by gamma distribution, modified gamma distribution, exponential distribution, normal distribution, lognormal distribution and so on[6,9,13–15]. The current commonly used model are exponential distribution[13], lognormal distribution[13] and gamma distribution[14–15].

$$n(D)=N_0\exp(-\Lambda D)\tag{12}$$

$$n(D)=\frac{N_0}{\sqrt{2\pi}\,\sigma D}\exp\left[-\frac{\ln^2(D/D^*)}{2\sigma^2}\right]\tag{13}$$

$$n(D)=N_0\frac{(ab)^{(2b-1)/b}}{\Gamma[(1-2b)/b]}D^{(1-3b)/b}\cdot\exp\left(-\frac{D}{ab}\right)\tag{14}$$

Where $n(D)$ is the volume concentration, D is particle dimension, N_0 is the total number of particles per unit volume, parameter Λ is positive that controls the curve change of the exponential distribution. standard deviation σ, mode radius D^*, effective radius a, gamma function Γ, the value of the effective variance b is between 0 and 0.5[14].

$W(D)$ is defined from Ref.[12], a probability density function for finding the particle size between D and $D+\mathrm{d}D$:

$$W(D)=\frac{n(D)}{N_0}\tag{15}$$

The statistical average DSS of ice cloud is given by

$$\langle\sigma_d\rangle=\int_0^{\infty}\sigma_d\cdot W(D)\mathrm{d}D\tag{16}$$

Substituting Eqs.(10) and (11) into Eq.(16), the statistical average DSS of static, moving ice cloud are obtained respectively by,

$$\langle\sigma_d\rangle=\int_0^\infty\left[\frac{S_1(\theta)^2\sin^2\phi+S_2(\theta)^2\cos^2\phi}{k^4r^2}\right]\cdot W(D)\mathrm{d}D \quad (17)$$

$$\langle\sigma_d\rangle=\int_0^\infty\left[\frac{S_1(\theta)^2\sin^2\phi+S_2(\theta)^2\cos^2\phi}{k^4r^2}\left(1+\frac{v_r^2}{\omega^2r^2}\right)\right]\cdot W(D)\mathrm{d}D \quad (18)$$

2 Numerical results and analysis

According to the Eqs. (6)–(9), the static and motion of ice sphere are considered. the DSS of spherical ice particle at three different laser wavelengths are numerically calculated. The complex refractive index of ice particle $m_1=1.308+i1.43\mathrm{e}-08$, $m_2=1.295\ 8+i1.31\mathrm{e}-05$, $m_3=1.290\ 6+i4.847\ 4\mathrm{e}-04$ corresponding to the incident laser wavelengths of 0.65, 1.31, 1.55 μm, respectively[16].

Corresponding to Eq.(10) and Eq.(11), the DSS of single static ice sphere are changed with their dimensions D and scattering angles θ at 0.65 μm wavelength are shown in Fig.2, the DSS of single moving ice sphere at v=100 m/s are shown in Fig.3. Comparison of Fig.2 and Fig.3, the calculated results are shown as following: (1) the DSS of single ice sphere is related strongly to its dimensions and scattering angles; (2) the DSS of static and of moving ice sphere are almost equal. That is, the moving velocity of spherical ice particle has little effect on the DSS of ice sphere.

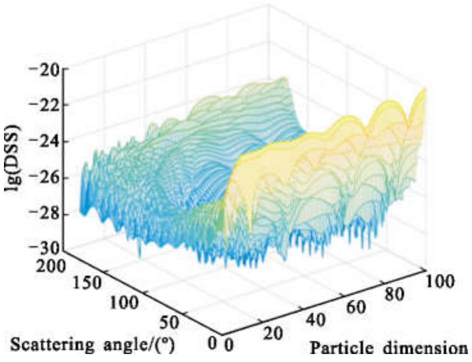

Fig.2 DSS of static ice sphere vs scattering angles and particle dimensions

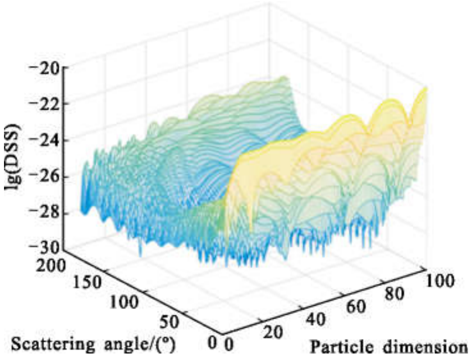

Fig.3 DSS of moving ice sphere vs scattering angles and particle dimensions

Figure 4 are shown that the DSS of single ice sphere are changed with their dimensions at 0.65, 1.31, 1.55 μm wavelengths, respectively. It is concluded that for different wavelengths, the trend of the DSS of ice sphere is different, and the local variation is also different.

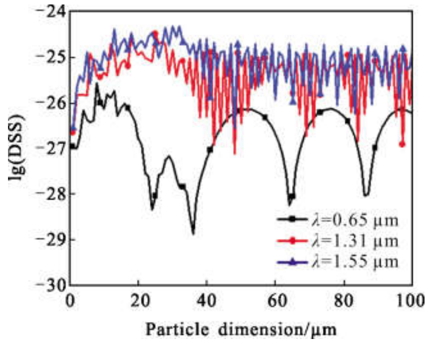

Fig.4 DSS of moving ice sphere vs particle dimension

Figure 5 are shown that the DSS of an ice sphere are changed with scattering angles at 0.65, 1.31, 1.55 μm wavelengths, respectively. The calculated results are shown that: (1) these scattering patterns with rapid

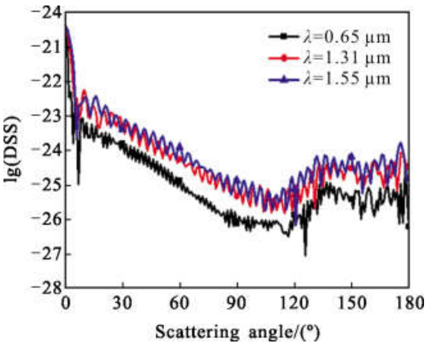

Fig.5 DSS of moving ice sphere vs scattering angle

fluctuation due to interference effects, depending on the size parameter; (2) the laser wavelength is longer, the DSS of ice sphere is larger.

Figure 6 are shown that the statistical average DSS of moving ice spheres with lognormal distribution changed with scattering angles at 0.65, 1.31, 1.55 μm wavelengths, respectively. It can be seen that the laser wavelength is longer, the statistical average DSS of ice spheres with rarefied random distribution is larger.

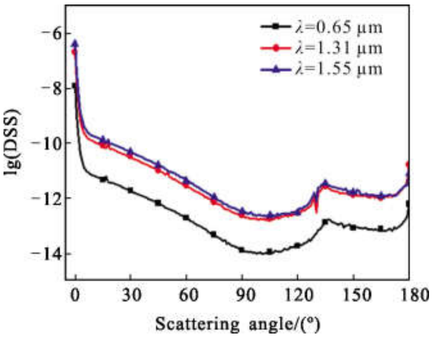

Fig.6 Average DSS of ice spheres with lognormal distribution vs scattering angle when particles are moving

Figure 7 are shown that the statistical average DSS of ice spheres with exponential distribution, lognormal distribution and gamma distribution changed with scattering angle at 1.55 μm under moving condition,respectively. The calculated results are shown that: (1) the trend of scattering patterns of ice spheres with different rarefied random distribution are almost the same, but there are differences at 138°;(2) average DSS of ice spheres with exponential distribution is the largest, which is several orders of magnitude larger than the other two distributions. It can be concluded that the size distribution model of ice particles has great influence on laser scattering properties of rarefied random distributed ice crystal particles.

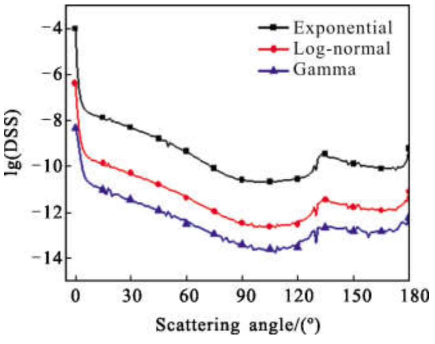

Fig.7 Average DSS of different rarefied random distributed ice spheres vs scattering angle when particles are moving

3 Conclusion

Research on the laser statistical scattering properties of ice crystal particles with different rarefied random distribution under moving condition are further improved in this paper. the DSS of single ice sphere changed with scattering angle and particle size are numerically calculated and analyzed at different laser wavelengths, and statistical average DSS of different rarefied random distributed ice spheres changed with scattering angle are compared. The numerical results show that:

(1) When the moving velocity of the ice sphere $\vec{v} \ll c$, the moving velocity of the ice sphere almost does not change the DSS of ice sphere;

(2) The laser wavelength is longer, the statistical average DSS of ice spheres with rarefied random distribution is larger;

(3) The particle size distribution models hardly change the trend of statistical average DSS of ice-crystals cloud, but significantly increases the order of magnitude of scattering patterns. Therefore, it can be concluded that the size distribution model of ice particles has great influence on laser scattering properties of rarefied random distributed ice crystal particles.

References:

[1] Liou K N, Yang P. Light Scattering by Ice Crystals: Fundamentals and Applications [M]. Cambridge: Cambridge University Press, 2016.

[2] Feigelson E M. Radiation in A Cloudy Atmosphere [M]. Netherland: Springer, 1984.

[3] Yang P, Hong G, Dessler A E, et al. Contrails and induced cirrus: Optics and radiation [J]. ***Bulletin of the American Meteorological Society***, 2010, 91(4): 473-478.

[4] Baran A J. From the single-scattering properties of ice crystals to climate prediction: A way forward [J]. ***Atmospheric Research***, 2012, 112: 45-69.

[5] Liu Dong, Liu Qun, Bai Jian, et al. Data processing algorithms of the space-borne lidar CALIOP: A review [J]. ***Infrared and Laser Engineering***, 2017, 46(12): 1202001. (in Chinese)

[6] Liao Zijun, Yang Chunping. Scattering properties of ice clouds in the visible light spectral region [J]. ***Optics & Optoelectronic Technology***, 2011, 9(6): 25-28. (in Chinese)

[7] Zhao Yanjie, Wei Heli, Xu Qingshan, et al. Simulation of radiative properties of ice particles at 1.315 μm[J]. ***Infrared and Laser Engineering***, 2009, 38(5): 782-786. (in Chinese)

[8] Yang P, Bi L, Baum B A, et al. Spectrally consistent scattering, absorption, and polarization properties of atmospheric ice crystals at wavelengths from 0.2 to 100 μm [J]. ***Journals of the Atmospheric Sciences***, 2013, 70(1): 330-347.

[9] Baum B A, Yang P, Heymsfield A J, et al. Ice cloud single-scattering property models with the full phase matrix at wavelengths from 0.2 to 100 μm[J]. ***Journal of Quantitative Spectroscopy & Radiative Transfer***, 2014, 146: 123-139.

[10] Bi L, Yang P. Improved ice particle optical property simulations in the ultraviolet to far-infrared regime [J]. ***Journal of Quantitative Spectroscopy & Radiative Transfer***, 2017, 189: 228-237.

[11] Wang Mingjun, Li Yingle, Wu Zhensen, et al. Laser scattering statistical characteristics of moving particles with rarefied random distribution [J]. ***Infrared and Laser Engineering***, 2011, 40(7): 1249-1253. (in Chinese)

[12] Ishimaru A. Wave Propagation and Scattering in Random Medium, Part I [M]. New York: Academic Press, 1978: 30-35.

[13] Tian L, Heymsfield G M, Heymsfield A J, et al. A study of cirrus ice particle size distribution using TC_4 observations[J]. ***Journal of the Atmospheric Sciences***, 2010, 67(1): 195-216.

[14] Emde C, Buras-Schnell R, Kylling A, et al. The libRadtran software package for radiative transfer calculations (version 2.0.1)[J]. ***Geoscientific Model Development***, 2016, 9(5): 1647-1672.

[15] Yi B, Yan P, Liu Q, et al. Improvements on the ice cloud modeling capabilities of the community radiative transfer model[J]. ***Journal of Geophysical Research***, 2016, 121(22): 1-14.

[16] Warren S G, Brandt R E. Optical constants of ice from the ultraviolet to the microwave: A revised compilation [J]. ***Journal of Geophysical Research Atmospheres***, 2008, 113 (D14): 762-770.